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Simulating mixed sea state waves for marine design

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ABSTRACT

This paper concerns the calculation of the wave statistics in nonlinear mixed sea states (i.e. sea states of combined wind waves and swell). The calculations have been carried out by incorporating a 2nd order nonlinear wave model into a random phase deterministic spectral amplitude (DSA) simulation method. To demonstrate the effectiveness of this approach, the calculation results from a linear simulation method and a random phase non-deterministic spectral amplitude (NSA) simulation method and from using some empirical formulas have also been presented for comparison purpose. In order to validate the approach proposed in this article, the wave characteristics directly calculated from the wave data measured at an oil field in the North Sea were utilized. It has been shown, contrary to the usual belief, that the nonlinear DSA simulation method used in this study can predict slightly more accurate wave characteristics than the nonlinear NSA simulation method does. The calculation results in this paper are analyzed, and some findings valuable for marine structural design have been pointed out.

1. Introduction

It is well known that not all sea states have unimodal wave spectra and narrow (or finite) spectral bandwidth. Frequently, sea states are due to the coexistence of various wave systems. In particular, local wind waves often develop in the presence of some background low frequency swell coming from distant storms, and the resulting mixed sea states will have bimodal wave spectra. In a sufficiently deep sea, the waves with bimodal wave spectra are generally considered to be a Gaussian random process. In the Gaussian sea model the individual cosine wave trains superimpose linearly (add) without interaction, and therefore the model is also called the linear sea model. In the existing literature, the linear sea model with bimodal wave spectra has been extensively studied and is relatively well understood [1]. However, it is known that for steep waves in deep water, or as the water depth decreases, the non-linearities become more and more relevant, and the ocean surface profile departs from the Gaussian assumption [1]. Under these conditions the wave profile becomes asymmetric, with higher and steeper crests, and shallower and flatter troughs, due to the interaction between individual cosine waves. Consequently, the linear Gaussian sea model can lead to incorrect predictions of wave characteristics in the actually nonlinear sea. In this paper, the wave characteristics in a nonlinear mixed sea state will be systematically investigated, and the specific characteristics studied will include the wave crest height distributions, wave crest height return values, wave height distributions, wave height and period joint distributions, and wave crest front steepness and height joint distributions.

Numerical simulation of random seas specified by a spectrum is performed by using one of the two basic approaches, i.e. superposition of harmonics and digital filtering of band-limited white noise. The first method is more established. It involves the superposition of a finite number of harmonics with different amplitudes, frequencies and a random phase [2,3]. The method of harmonic superposition has been the most widely used because of its simplicity and the capability of maintaining spatial correlations. The harmonic superposition model can be further subdivided into the random phase deterministic spectral amplitude (DSA) model, and the non-deterministic spectral amplitude (NSA) model [4]. In the DSA method the sample spectrum will be identical to the target spectrum for every simulation. On the contrary, the sample spectrum of time histories simulated by the NSA method will have a random fluctuation about the target spectrum, each sample spectrum being different from the other. Wang et al. [5] and Wang et al. [6] utilized the DSA method to simulate the water particle velocity and acceleration stochastic processes in the Morison equation for calculating the response statistics of a Tension Leg Platform. This simulation work was performed in the symbolic manipulation program Mathematica and summarized in Wang and Tan [7]. Wang [8] and Wang and Tan [9] utilized the DSA method to calculate the slow drift surge extreme responses of a moored floating cylinder. Wang [8] and Wang and Tan [9] also proposed a compound path integral

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solution method in order to further improve the efficiency of a path integral solution method for predicting nonlinear system responses under random excitations. The idea of the compound path integral solution method is to first estimate a rough initial probability density by using small number of DSA simulations and the 3 sigma rule of Normal Distribution.

In this article, the calculation of the wave characteristics in nonlinear mixed sea states will be performed. The calculations will be carried out by incorporating a 2nd order nonlinear wave model into a DSA simulation method. To demonstrate the effectiveness of this approach, the calculation results from a linear simulation method and a nonlinear NSA simulation method and from using some empirical formulas will also be presented for comparison purpose. In order to validate the approach proposed in this article, the wave characteristics directly calculated from the wave data measured at the Gullfaks C platform in the North Sea (http://www.maths.lth.se/matsta/wafo/documentation/wafodoc/wafo/data/gfaksr89.html) will be utilized. It will be shown, contrary to the usual belief, that the nonlinear DSA simulation method used in this study can predict slightly more accurate wave characteristics than the nonlinear NSA simulation method does. The calculation results in this paper will be analyzed, and some findings valuable for marine structural design will be pointed out.

2. Second order nonlinear waves in mixed sea states

The nonlinear mixed sea states can be obtained by adding to the linear Gaussian sea model "second order" correction terms allowing for interactions between the elementary sinusoidal waves. A second order model for the wave elevation $\eta(t)$ at a fixed spatial location can thus be written as:

$$\eta(t) = \eta_G(t) + \eta_2(t) \tag{1}$$

where $\eta_G(t)$ is the linear Gaussian process, $\eta_2(t)$ is the second order correction process from the nonlinear hydrodynamic problem associated with waves, and t denotes time. The standard Fourier sum for the linear Gaussian process is given by [10]:

$$\eta_G(t) = \sum_{k=1}^{N} A_k \cos(\omega_k t - \theta_k) = Re \sum_{k=1}^{N} C_k \exp(i\omega_k t) \tag{2}$$

where for each elementary sinusoidal wave A_k denotes its amplitude, ω_k the angular frequency, and θ_k the phase angle. In the above formula, Re indicates the real part of a complex number, and C_k's are mutually independent of one another.

Based on Volterra theory, the second order correction is given by:

$$\eta_2(x, t) = Re \sum_{m=1}^{N} \sum_{n=1}^{N} C_m C_n \left[H_{mn}^{+} e^{i(\omega_m+\omega_n)t} + H_{mn}^{-} e^{i(\omega_m-\omega_n)t} \right] \tag{3}$$

where the sum-frequency quadratic transfer function H_{mn}^{+} in a finite water depth d is given by [10]:

$$H_{mn}^{+} = \frac{g((k_m k_n)/(\omega_m \omega_n)) - (1/(2g))\left(\omega_m^2 + \omega_n^2 + \omega_m \omega_n\right) + (g/2)\left(\left(\omega_m k_n^2 + \omega_m k_n^2\right)/(\omega_m \omega_n(\omega_m + \omega_n))\right)}{1 - g((k_m + k_n)/(\omega_m + \omega_n)^2)\tanh((k_m + k_n)d)} - g\frac{k_m k_n}{2\omega_m \omega_n} + \frac{1}{2g}\left(\omega_m^2 + \omega_n^2 + \omega_m \omega_n\right) \tag{4}$$

where the wave numbers k_n and circular frequencies ω_n are related through the linear dispersion relation:

$$\omega_n^2 = g k_n \tanh(k_n d) \tag{5}$$

where g and d are the gravitational acceleration and water depth, respectively. The corresponding difference-frequency quadratic transfer function H_{mn}^{-} is found by replacing ω_n by $-\omega_n$ and k_n by $-k_n$.

3. The numerical simulation methods

The second order model for the wave elevation $\eta(t)$ can be simulated by the superposition of harmonics with random phases.

The harmonic superposition model can be further subdivided into two methods. The first involves deterministic amplitudes and random phases. The second involves non-deterministic amplitude. In the next, the theoretical background of these two methods is briefly introduced.

3.1. *The random phase deterministic spectral amplitude (DSA) method*

Consider a 1D–1V stationary stochastic process $\eta_0(t)$ with mean value equal to zero, autocorrelation function $R_{\eta_0\eta_0}(\tau)$ and one-sided power spectral density function $S_{\eta_0\eta_0}(\omega)$. The stochastic process $\eta_0(t)$ can be simulated by the following series as $N \to \infty$ [11]:

$$\eta(t) = \sqrt{2} \sum_{k=1}^{N} A_k \cos(\omega_k t - \theta_k) \tag{6}$$

where

$$A_k = (S_{\eta_0\eta_0}(\omega_k)\Delta\omega)^{1/2}, \quad k = 1, 2, \ldots N \tag{7}$$

$$\omega_k = k\Delta\omega, \quad k = 1, 2, \ldots N \tag{8}$$

$$A_1 = 0. \tag{9}$$

Or

$$S_{\eta_0\eta_0}(\omega_0 = 0) = 0. \tag{10}$$

The $\theta_1, \theta_2, \theta_3, \ldots \theta_k$ appearing in Eq. (6) are independent random phase angles distributed uniformly over the interval $[0, 2\pi]$. A sample function $\eta^i(t)$ of the simulated stochastic process $\eta(t)$ can be obtained by replacing the sequence of random phase angles θ_1, $\theta_2, \theta_3, \ldots \theta_k$ with their respective i-th realizations ${\theta_1}^{(i)}, {\theta_2}^{(i)}, {\theta_3}^{(i)}, \ldots {\theta_k}^{(i)}$ [11]:

$$\eta^i(t) = \sqrt{2} \sum_{k=1}^{N} A_k \cos\left(\omega_k t - {\theta_k}^{(i)}\right) \tag{11}$$

3.2. *The non-deterministic spectral amplitude (NSA) method*

The non-deterministic spectral amplitude (NSA) method is expressed as [4]:

$$\eta(t) = \sum_{k=1}^{N} [A_k \cos(\omega_k t) + B_k \sin(\omega_k t)] \tag{12}$$

The coefficients A_k and B_k are independent random variables with zero mean and a standard deviation given by

$$\sigma_{A_k} = \sigma_{B_k} = (2S(\omega_k)\Delta\omega)^{1/2} \tag{13}$$