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Computational uncertainty quantification for a clarifier-thickener model with several random perturbations: A hybrid stochastic Galerkin approach

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ABSTRACT

Continuous sedimentation processes in a clarifier-thickener unit can be described by a scalar nonlinear conservation law whose flux density function is discontinuous with respect to the spatial position. In the applications of this model, which include mineral processing and wastewater treatment, the rate and composition of the feed flow cannot be given deterministically. Efficient numerical simulation is required to quantify the effect of uncertainty in these control parameters in terms of the response of the clarifier-thickener system. Thus, the problem at hand is one of uncertainty quantification for nonlinear hyperbolic problems with several random perturbations. The presented hybrid stochastic Galerkin method is devised so as to extend the polynomial chaos approximation by multiresolution discretization in the stochastic space. This approach leads to a deterministic hyperbolic system, which is partially decoupled and therefore suitable for efficient parallelisation. Stochastic adaptivity reduces the computational effort. Several numerical experiments are presented.

1. Introduction

1.1. Scope

In many technical applications one seeks to quantify the stochastic variability of the response of a nonlinear system, usually defined in terms of a partial differential equation (PDE), with respect to uncertainty in initial conditions, control parameters and coefficient functions. This uncertainty can be quantified by aleatoric variation of parameters and sampling corresponding solutions in a "Monte Carlo (MC)"-like manner. This method is easy to implement but very inefficient due to the slow convergence in the sampling variable. One therefore prefers deterministic models for at least a finite number of stochastic moments to quantify randomness (an overview is given by Matthies and Keese (2005)). For instance, stochastic Galerkin (SG) or collocation methods seem to be more promising to handle the present situation. This approach is well understood for models governed by linear PDEs. We herein focus on nonlinear problems posed by hyperbolic conservation laws, and thereby complement recent efforts in uncertainty quantification for nonlinear problems considered by Poëtte et al. (2009), Tryoen et al. (2010, 2012), and Abgrall and Congedo (2013).

It is the purpose of this paper to extend the hybrid stochastic Galerkin (HSG) discretization introduced by Bürger et al. (2014), Köppel et al. (2014), and Kröker et al. (2015) to several stochastic dimensions, and to apply it to a model governed by a scalar, nonlinear hyperbolic conservation law. Specifically, we consider a clarifier-thickener (CT) model for the continuous solid-liquid separation of suspensions under gravity discussed by Bürger et al. (2004, 2012a), Betancourt et al. (2013), Diehl (1996a, 2001, 2005, 2006, 2008) and Nocoń (2006), see Fig. 1. Bürger et al. (2005) introduced this strongly idealised description of secondary settling tanks in wastewater treatment or thickeners in mineral processing. For suspensions of small solid particles in a fluid, the governing PDE is a first-order scalar conservation law with a flux density function that depends spatially on position. For the discussion on the well-posedness and numerical analysis of this equation we refer to Chancelier et al. (1994), Diehl (1996a,b, 2001, 2005, 2006, 2008) and Bürger et al. (2004, 2005).

In the clarifier-thickener and related multiphase flow models, many input parameters cannot be described with deterministic accuracy but behave stochastically. For instance, in mineral processing the uncertainty comes from the fact that the feed flow stems from other units that are not under control of the

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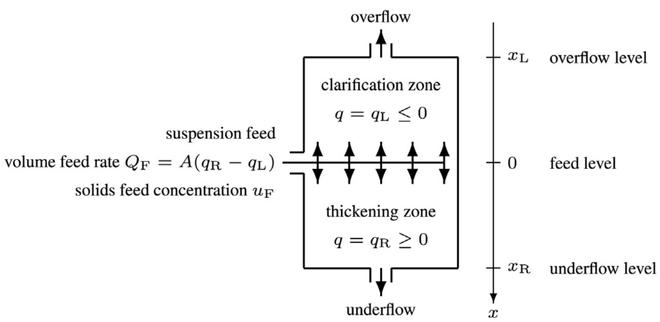

Fig. 1. Principle of the clarifier-thickener (CT) model. The cylindrical unit of cross-sectional area A occupies the depth interval $[x_\mathrm{L}, x_\mathrm{R}]$. Suspension to be separated is fed at level $x = 0$ at rate Q_F and concentration u_F. The feed flow is split into upward- and downward-directed bulk flows with respective velocities $q_\mathrm{L} \leq 0$ and $q_\mathrm{R} \geq 0$. In the deterministic setting, u_F, Q_F and q_R, and therefore also q_L, are known control parameters. Under normal circumstances, concentrated sediment forms on the bottom of the thickening zone and is continuously removed with the underflow while clarified liquid laves the unit with the overflow. It is assumed that the solid-liquid separation takes place within the unit only, identified by the x-interval $(x_\mathrm{L}, x_\mathrm{R})$, while outside, in the overflow and underflow streams, both phases move at the same velocity.

CT operator, while in wastewater treatment weather conditions, which may affect the operation of the unit, are unpredictable. Bürger et al. (2014) analysed the effect of uncertainty in the feed concentration u_F. This uncertainty produces a first-order scalar conservation law with a random flux function. In this work we provide a new efficient method for evaluating the uncertainty of the response of the system, that is, of the exact or numerical solution of the governing PDE, in terms of the uncertainty in *three* control parameters, namely u_F and the so-called bulk flows, denoted by q_L and q_R. (Uncertainty in q_L and q_R can equivalently be expressed as uncertainty in the volume feed flow, equivalent to $q_\mathrm{R} - q_\mathrm{L}$, and in CT control actions, which are expressed by q_R.)

To provide further justification of our approach we mention that utilizing spatially one-dimensional descriptions of clarifier-thickener units (known as *secondary settling tanks* (SSTs) in wastewater treatment) is common practice in engineering applications. For simulation, design, and control computation one wishes to avoid the necessity to compute a two- or three-dimensional flow field since this invariably incurs the necessity to solve additional equations, namely versions of the Stokes or Navier–Stokes equations, to determine the flow field for the mixture. However, experimental observations, numerical simulations and practical considerations (Ekama and Marais, 2004; Bürger et al., 2015) indicate that for moderate to high concentrations the solids concentrations is nearly horizontally constant, and that the sedimentation process can be captured by a one-dimensional (vertical) model. In fact, one-dimensional models that at least for the special case of non-flocculated suspensions are equivalent to the present approach were studied among others by Abusam and Keesman (2009), Chancelier et al. (1994), David et al. (2009a,b), Diehl (2001, 2005, 2006, 2008), Guo et al. (2010), Li and Stenstrom (2015), Nocoń (2006), Plósz et al. (2007), and Verdickt et al. (2005). (This list is far from being complete, and we here limit ourselves to works by alternate authors.) Furthermore, works that explicitly address models of this type to analyze the system response to stochastic variations of input parameters, and possible strategies to control the system behaviour, include those by Betancourt et al. (2013), Guyonvarch et al. (2015) and Torfs et al. (2015). In light of all these references the model and numerical methods presented for its solution herein correspond to the state of the art and widespread practice in clarifier-thickener modeling, even though the process model is in itself a simplification of a real-world clarifier-thickener.

As a classical approach in random perturbed PDEs one could apply the SG method for uncertainty quantification. This method is based on representing the random field by a truncated sum of orthonormal polynomials. Poëtte et al. (2009) showed that this approach leads to a very accurate approximation in terms of a strongly coupled, high-dimensional deterministic system for a finite number of moments. Tryoen et al. (2012) presented an alternative approach obtained by the multi-wavelet stochastic discretization, which still leads to a full coupling of the polynomial basis that is defined on the whole stochastic domain. Our approach consists in a combination of both concepts, namely we devise a hybrid stochastic Galerkin (HSG) method that combines polynomial chaos (PC) and multi-wavelet representations. As a consequence, each stochastic element is equipped with its own polynomial basis. This combination has the decisive advantage that the HSG method leads to a partially decoupled deterministic system that allows efficient parallelization. Furthermore, we improve the efficiency of the HSG method by adaptive multiresolution in the stochastic space (we also refer to Le Maître et al. (2004) and Bürger et al. (2014) for adaptive approaches in the framework of multi-resolution techniques).

1.2. Outline of the paper

The remainder of the paper is organized as follows. In Section 2 the governing model is described. To this end, we summarize in Section 2.1 a deterministic, spatially one-dimensional CT model (cf. Fig. 1). In Section 2.2 we include the random perturbations. In Section 3 we introduce an approximation for the random perturbations by the SG and the new HSG approaches. Specifically, we review the PC approach in Section 3.1 and define in Section 3.3 the SG system. This leads after a finite volume discretization in the one-dimensional physical space to the stochastic Galerkin finite volume (SG-FV) method. In Section 3.4 we extend the SG stochastic discretisation to the HSG approach. In Section 3.4.1 we explain how the coefficients of the HSG representation are calculated. Next, in Section 3.4.2 the HSG approach is extended to several stochastic dimensions and in Section 3.4.3 applied to the clarifier-thickener model. A fully discrete finite volume formulation for the SG approaches, namely the respective "HSG-system" is introduced in Section 3.4.4 (HSG-FV). The further improvement of the method is stochastic adaptivity (denoted as HSG-adapt), which is introduced in Section 4. The HSG-adapt method reduces the stochastic dimension and increases the computational efficiency decisively. Section 4 starts with a short description of properties of the multi-wavelet basis in Section 4.1 and proceeds with the extension to the multivariate case in Section 4.2. In Section 4.3 we recapitulate the concept of the graded tree and introduce an N_r-adaptivity algorithm for the HSG-discretization based on this concept. Section 5 is devoted to the presentation of numerical examples. In Section 5.1 we present experiments in two and three stochastic dimensions and compare the HSG-FV results with those of a Monte Carlo approach. In Section 5.2 we discuss the benefits of the parallel application for HSG methods. The numerical experiments in Section 5.3 confirm the efficiency and accuracy of the HSG-adapt method. In Section 6 we present the application of the HSG approach to the real world problem. Conclusions of the paper are summarized in Section 7.

2. Governing models

2.1. Deterministic version

The CT model is based on the conservation equations of the solid and the fluid. Both are considered as superimposed continuous phases with velocities v_s and v_f, respectively. In terms of the solid-fluid relative velocity $v_\mathrm{r} := v_\mathrm{s} - v_\mathrm{f}$ and the volume-average velocity