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Wave power statistics for individual waves

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ABSTRACT

The paper provides a bivariate distribution of wave power and wave height, as well as a bivariate distribution of wave power and wave period; both bivariate distributions are for individual waves within a sea state. This is relevant for e.g. making assessments of wave power devices and their potential for converting energy from waves. The results can be applied to compare systematically the wave power potential for individual waves in a given sea state at different locations.

1. Introduction

Ocean wave energy appears to be promising as a source of alternative energy. The design of appropriate devices to convert energy from waves is a challenge for the engineering community. The response of a wave energy device is generally frequency dependent, i.e. the resonance frequency and the frequency range which will give a significant response, will depend on the design of the device. A device will be exposed to a wide range of sea states characterized by the significant wave height H_s and a characteristic wave period, i.e. the spectral peak period T_p or the mean zero-crossing wave period T_z. Moreover, within a sea state, the individual random waves are characterized by the wave height H and the wave period T.

A requirement for the optimum design of a wave power device is that it can be controlled to produce maximum wave power at a low cost in a sea state. At the same time the control should protect the device when exposed to extreme waves. This requires that its response properties can be changed to match the change in wave conditions. Typically this means that its resonance period is changed to match the characteristic wave period of the sea state. For a given sea state it should also be possible to control the device to resonate with the most energetic waves, or with waves having periods above a threshold value; see e.g. [1,2]. A recent review of wave energy extraction is given in [3].

Thus, a knowledge of the statistical properties of the waves is crucial for designing a proper wave power device, i.e. both for the sea state and the single random waves within a sea state. It is also of interest to know e.g. the joint statistical properties of the wave power in a sea state with significant wave height or a characteristic wave period, or the joint statistics of the wave power for individual waves with the individual wave height or the individual wave period. Some examples of previous work on wave power statistics for sea states are [4–7], while Smith et al. [8] and Venugopal and Smith [1] studied the wave power statistics for individual waves.

The purpose of the present paper is to provide bivariate distributions of wave power with wave height and wave power with wave period for individual waves within a sea state, and to discuss statistical aspects of wave power for individual waves. The results can be applied to compare systematically the wave power potential in different sea states at different locations based on short term statistical description of the waves.

2. Background

The wave power is defined as the transport of wave energy per unit crest length of the progressive wave front, which for waves in deep water is given by (see e.g. [9])

$$J = \frac{\gamma g^2}{32\pi} H^2 T. \tag{1}$$

Here γ is the fluid density, g is the acceleration of gravity, H is the wave height, and T is the wave period. In a sea state of random waves Eq. (1) can be taken to represent the wave power associated with a single random wave with wave height H and wave period T. Different models of the joint probability density function (*pdf*) of H and T are given in the literature. Examples are: [10–13].

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Comparisons of distributions with observed wave data have been presented by e.g. [14,15].

In the present paper the [13] (hereafter referred to as MK84) distribution is chosen to serve the purpose of demonstrating how a joint *pdf* of H and T can be used to provide the statistical properties of J. MK84 was derived by fitting a parametric model to data obtained from measurements at sea on the Norwegian continental shelf including altogether 6353 individual zero-downcross waves. These data were taken from a larger data base sampled with three Waverider buoys located at three different deep water locations. The MK84 model is given as

$$p(h,t) = p(t|h)p(h) \tag{2}$$

where the marginal *pdf* of h is given by the 2-parameter Weibull *pdf*

$$p\,(h) = \frac{2.39h^{1.39}}{1.05^{2.39}} \exp\left[-\left(\frac{h}{1.05}\right)^{2.39}\right]; \quad h \geq 0 \tag{3}$$

and the conditional *pdf* of t given h is given by the 3-parameter Weibull *pdf*

$$p(t|h) = \frac{\beta}{\rho}\left(\frac{t-\alpha}{\rho}\right)^{\beta-1} \exp\left[-\left(\frac{t-\alpha}{\rho}\right)^{\beta}\right]; \quad t \geq \alpha \tag{4}$$

with the parameters

$$\alpha = 0.12\sqrt{h} \tag{5}$$

$$\rho = \begin{cases} 0.78h + 0.26 & \text{for } h \leq 0.9 \\ 0.962 & \text{for } h > 0.9 \end{cases} \tag{6}$$

$$\beta = 2\,arctg\ [2\,(h-1.2)] + 5. \tag{7}$$

Here $h = H/H_{rms}$ and $t = T/T_{rms}$ are the normalized variables made dimensionless by using the root-mean-square (*rms*) values. For these data the *rms* values are related to the significant wave height H_s and the mean zero-crossing wave period T_z by, respectively,

$$H_{rms} = 0.714H_s \tag{8}$$

and

$$T_{rms} = 1.2416T_z. \tag{9}$$

Here Eqs. (8) and (9) are obtained as the best fit to data by linear regression analysis. Since H_{rms} and T_{rms} are constants for each sea state, $p(h,t)$ in Eqs. (2)–(7) is a conditional *pdf* given a sea state, i.e. given H_{rms} and T_{rms} (or equivalently H_s and T_z).

By introducing the non-dimensional wave power $j = J/J_{char}$, Eq. (1) can be re-arranged to

$$j = h^2 t \tag{10}$$

where

$$J_{char} = \frac{\gamma g^2}{32\pi} H_{rms}^2 T_{rms} \tag{11}$$

is a characteristic wave power for the sea state.

3. Statistical properties of wave power

Statistical properties of the non-dimensional wave power j (from which the statistical properties of the wave power J can be obtained) can be derived by using the joint *pdf* of h and t, e.g. giving the joint *pdf* of j and h. First, the joint *pdf* of j and h is obtained from Eq. (10) by a change of variables from (h,t) to (h,j), which takes the form

$$p\,(h,j) = p\,(j|h)\,p\,(h)\,. \tag{12}$$

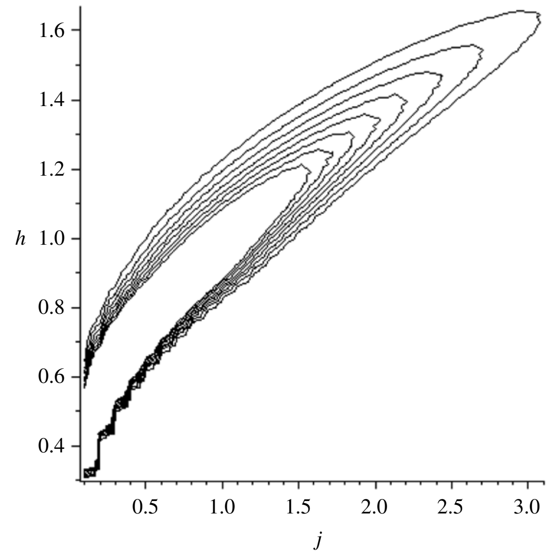

Fig. 1. Isocontours of $p\,(h,j)$. The levels of the eight outer contours from the outermost contour are 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9.

It should be noted that this change of variables only affects $p\,(j|h)$ since $t = j/h^2$, yielding a 3-parameter Weibull *pdf* of j given h in the form

$$p\,(j|h) = \frac{\beta}{\rho h^2}\left(\frac{j-\alpha h^2}{\rho h^2}\right)^{\beta-1} \exp\left[-\left(\frac{j-\alpha h^2}{\rho h^2}\right)^{\beta}\right]; \quad j \geq \alpha h^2. \tag{13}$$

The joint *pdf* of t and j is obtained from Eq. (10) by a change of variables from (h,t) to (t,j), which takes the form (by using the Jacobian $|\partial j/\partial h| = 2ht$)

$$p\,(j,t) = \frac{p\,(h,t)}{2ht} = \frac{p\,(t|h)\,p\,(h)}{2ht} = p\left(t|h = \frac{\sqrt{j}}{\sqrt{t}}\right) p\,(j;t) \tag{14}$$

where

$$p\,(j;t) = \frac{p\,(h)}{2ht} = p\left(h = \frac{\sqrt{j}}{\sqrt{t}}\right) \Big/ 2\sqrt{j}\sqrt{t}. \tag{15}$$

It should be noted that the notation in Eq. (15) is used to make it clear that $p(j;t)$ depends explicitly on t. By substitution in Eq. (3), it follows that Eq. (15) can be re-arranged to the 2-parameter Weibull *pdf*

$$p\,(j;t) = \frac{s}{r}\left(\frac{j}{r}\right)^{s-1} \exp\left[-\left(\frac{j}{r}\right)^{s}\right]; \quad j \geq 0 \tag{16}$$

with the parameters

$$r = 1.05^2 t \tag{17}$$

$$s = 2.39/2. \tag{18}$$

Moreover, $p\left(t|h = \sqrt{j}/\sqrt{t}\right)$ is given by Eqs. (4)–(7) to by substituting for h, i.e. $h = \sqrt{j}/\sqrt{t}$.

Fig. 1 shows the isocontours of $p\,(h,j)$, and the quadratic increase of j with h is clearly seen. The peak value of the *pdf* is $p_{\max} = 6.2$ and is located at $h = 0.41$ and $j = 0.1$, i.e. at a rather low value of h. This reflects the location of the peak of the MK84 *pdf* of h and t (see MK84, Fig. 17).

Fig. 2 shows the cumulative distribution function (*cdf*) of j, and is based on integration of the joint *pdf* in Fig. 1. It appears that there is a factor of approximately 3.2 between the 50% ($j = 0.76$) and the 90% ($j = 2.4$) fractiles. In this case the relatively narrow shape of the joint *pdf* which can be observed in Fig. 1 is reflected also here.