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Extending a time/frequency domain hybrid method for riser fatigue analysis

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ABSTRACT

Fully coupled time domain analysis of a floating system is rigorous but time-consuming, thus an efficient hybrid method was developed in a prior work. The method simulates the coupled slow-drift responses in the time domain and analyzes the wave-frequency dynamics in the frequency domain. The hybrid method was shown to compare well with time domain analysis in predicting the mean-square vessel motions and line tensions. In this paper, the hybrid method is extended for the fatigue analysis of moorings and risers. The fatigue damage thus calculated is found to closely agree with the result from time domain analysis in conjunction with rainflow counting. Several spectral fatigue techniques are also compared, and these methods are significantly less accurate, thus attesting to the complexity of the problem.

1. Introduction

In the design of a floating production system, it has been the tradition to undertake the dynamic analysis of the platform and the connected moorings and risers separately, and this approach is known as an uncoupled analysis. In an uncoupled analysis, the dynamic effects of the lines (moorings and risers) on the vessel are neglected. Over the past decade, many studies (e.g. [1–3]) have shown that in deepwater, the line dynamics can have pronounced influence on the platform motions, and consequently, coupled analysis has come to the forefront. A coupled analysis entails a simultaneous analysis of the platform and the attached moorings and risers in order to rigorously account for all the dynamic interactions.

Although coupled analysis in the time domain is accurate and reliable, the associated computational costs are prohibitive for routine work for at least two reasons. First, the full global system is modeled by many degrees-of-freedom. Second, a small time step is required to capture the system response at the wave frequency (WF), whereas a lengthy simulation duration is necessitated in order to ensure sufficient cycles for the low frequency (LF) response. Due to practical requirements, a number of efficient strategies have been proposed. By far the most common technique is to estimate the effects of inertia and damping from the lines as linear coefficients which are then applied to an uncoupled analysis of the platform [1,4]. Nevertheless, this methodology is not without drawbacks; vessel motions at the fairlead are necessary to obtain the damping estimates, but in reality these motions are themselves dependent on the damping levels [5]. Moreover, an equivalent damping coefficient, however precisely estimated, may not be adequate, as the damping is inevitably highly complicated owing to the nonlinearities from line geometry and the drag force. In particular, the damping levels are strongly dependent on the vessel excursion.

To circumvent some of the limitations inherent in existing strategies, Low and Langley [6] developed a hybrid time/frequency domain method, aimed at solving the coupled problem accurately and yet in an efficient manner. In this *hybrid method*, the LF response of the complete coupled floating system is simulated in the time domain. Intermittently, the global model is transferred onto a separate module to analyze the WF response in the frequency domain. The frequency domain then returns specific information to the time domain program. The feedback mechanism enables the coupling between the responses at the two time-scales to be maintained. Compared to a fully coupled time domain analysis, the hybrid method consumes an order of magnitude less computational time, yet the accuracy is comparable in terms of predicting the mean-square responses. This is true even for a system with pronounced geometric nonlinearity caused by changes in the line configuration.

The designer is faced with two important criteria: the ultimate limit state and the fatigue limit state. In both situations, the number of load cases to be analyzed is considerable, and this is where the benefits of having an efficient tool such as the hybrid method become striking. There are established procedures for computing the extreme responses and the fatigue damage from conventional time domain or frequency domain data. However, the results produced by the hybrid method is in a unique format;

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the time history is available for the LF vessel motions and line loads, whereas the WF response is described statistically in the form of its spectral moments. In order to fully exploit the available information to arrive at the most accurate result for the limit states, specially developed algorithms are essential, since those in the literature are inapplicable. In a sequel [7], the hybrid method was extended for predicting the extreme response.

Fatigue analysis is an integral aspect in the design of moorings and risers. A floating system experiences a multitude of different sea states (mild, moderate and severe) during its lifetime, and the riser fatigue assessment should be based on the aggregate of the load effects from the various sea states. The accepted procedure is to employ a scatter diagram of the wave height and wave period to describe the long term statistics of the environmental condition. As the number of bins in a scatter diagram is typically excessive, a common practice [8,9] is to condense the sea state data into a smaller quantity of representative blocks that are manageable. The main contribution to the total fatigue damage usually results from low to moderate sea states with a high probability of occurrence, instead of a handful of extreme conditions. For this reason, it is common to disregard the extreme environmental conditions in fatigue design. The main strength of the hybrid method is its ability to cope with severe nonlinearities. However, mild sea states are typically associated with weak nonlinearities, in which case a linearized frequency domain approach will generally suffice. Thus, the hybrid method is targeted for the moderate sea states, in which nonlinearities may be consequential.

The aim of this paper is to devise a technique for evaluating the fatigue performance. The accuracy of the fatigue damage prediction is benchmarked against the result from fully coupled time domain analysis, in which the rainflow counting algorithm is used. There are a number of complicating features that make the aforementioned task an interesting and challenging one. First, the statistics of the line tension are non-Gaussian for both the LF and WF components. Second, the WF tension statistics are non-stationary, since they depend on various factors, such as the vessel offset. Last, the WF and LF parts of the tension are strongly coupled.

2. Background

2.1. Summary of the hybrid method

An overview of the hybrid method is provided in this subsection; details may be found in [6]. The hybrid method builds on in-house computer programs, comprising several modules that are capable of performing coupled/uncoupled analysis of the vessel and lines in both the time and frequency domain. The platform is modeled as a rigid body with six degrees-of-freedom, and the lines are discretized by the lumped mass approach. The position-dependent added mass and tangent stiffness matrices are formulated in global coordinates. A set of stiff springs connect the top node of each line to a point on the vessel. Seabed interaction is described using a bilinear spring model. The first- and second-order wave forces on the vessel are solved from diffraction analyses, while the three-dimensional fluid forces acting on the lines are calculated from Morison's equation.

The principle of the hybrid method is to perform a time domain analysis of the coupled system that includes only the second-order LF forces. Since the response varies slowly, a large time step is feasible. At predetermined regular intervals, say a fifth of a LF cycle, the time domain analysis is momentarily halted. Specific information, including the instantaneous position (characterized by the mass and stiffness matrices) and the LF relative velocities of the line nodes, is transferred onto a separate module, in which the first-order WF response is solved in the frequency domain.

In both the time and frequency domain modules, the drag force is linearized separately in two orthogonal directions. Since the LF velocity, denoted as $\dot{y}^{(2)}$, is slowly varying, it is treated as an additional current for linearization purposes. In an arbitrary direction, the actual current is represented as V_c, and the first- and second-order relative velocities as $\upsilon^{(1)}$ and $\upsilon^{(2)}$. Because wave kinematics are invalid for the LF part, $\upsilon^{(2)} = -\dot{y}^{(2)}$. The velocity-squared expression within the Morison drag force is linearized as

$$(V_c - \dot{y}^{(2)} + \upsilon^{(1)})|V_c - \dot{y}^{(2)} + \upsilon^{(1)}| = \beta\upsilon^{(1)} + \gamma \tag{1}$$

where

$$\beta = \sqrt{\frac{8}{\pi}}\sigma_{\upsilon(1)} \exp\left[-\frac{1}{2}\left(\frac{\hat{V}}{\sigma_{\upsilon(1)}}\right)^2\right] + 2\hat{V}\mathrm{erf}\left(\frac{\hat{V}}{\sqrt{2}\sigma_{\upsilon(1)}}\right) \tag{2}$$

$$\gamma = \sqrt{\frac{2}{\pi}}\hat{V}\sigma_{\upsilon(1)} \exp\left[-\frac{1}{2}\left(\frac{\hat{V}}{\sigma_{\upsilon(1)}}\right)^2\right] + (\sigma^2_{\upsilon(1)} + \hat{V}^2)\mathrm{erf}\left(\frac{\hat{V}}{\sqrt{2}\sigma_{\upsilon(1)}}\right) \tag{3}$$

$$\hat{V} = V_c - \dot{y}^{(2)} \tag{4}$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \mathrm{e}^{-y^2}\mathrm{d}y. \tag{5}$$

The term γ corresponds to the mean drag force and is not explicitly considered in the frequency domain portion of the hybrid method, since the mean position is already known from the time domain counterpart. Instead, it is used to define the instantaneous LF drag force in the time domain analysis, in which $\sigma_{\upsilon(1)}$ in Eq. (4) is unaltered up till the subsequent frequency domain analysis, but $\dot{y}^{(2)}$ is continually updated.

The purpose of the frequency domain analysis is twofold. First, $\sigma_{\upsilon(1)}$ is fed back to the time domain analysis so that the WF and LF responses remain coupled. Second, the non-stationary statistics of the WF line tensions are continually reported in the form of the spectral moments

$$\lambda_i^{(1)} = \int_0^\infty \omega^i S_{XX}^{(1)}\mathrm{d}\omega, \quad i = 0, 1, 2, \ldots \tag{6}$$

where $S_{XX}^{(1)}$ is the one-sided spectrum of the WF line tension $X^{(1)}$.

The motivation for the hybrid scheme is as follows. The LF motions are non-Gaussian and can be quite substantial, thus activating the geometric nonlinearity of the lines. Moreover, the LF motions are sensitive to the drag-induced damping, which is time-varying as it depends on various factors such as the vessel offset. These characteristics demand a fully nonlinear time domain approach. In contrast, the WF oscillations about a given offset are typically small, and are nominally linear and Gaussian, thus a frequency domain analysis will often suffice. Although the WF line tensions are moderately non-Gaussian, this feature can be accommodated in an approximate fashion, as will be elaborated shortly.

2.2. Fatigue assessment method

In fatigue analysis, the stress range S and the associated number of cycles to failure N is commonly described using an S–N curve. A single-slope S–N curve has the exponential form

$$N = KS^{-m} \tag{7}$$

where K and m are material parameters determined from laboratory tests.