



# An MILP-MINLP decomposition method for the global optimization of a source based model of the multiperiod blending problem



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## ABSTRACT

The multiperiod blending problem involves binary variables and bilinear terms, yielding a nonconvex MINLP. In this work we present two major contributions for the global solution of the problem. The first one is an alternative formulation of the problem. This formulation makes use of redundant constraints that improve the MILP relaxation of the MINLP. The second contribution is an algorithm that decomposes the MINLP model into two levels. The first level, or master problem, is an MILP relaxation of the original MINLP. The second level, or subproblem, is a smaller MINLP in which some of the binary variables of the original problem are fixed. The results show that the new formulation can be solved faster than alternative models, and that the decomposition method can solve the problems faster than state of the art general purpose solvers.

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## 1. Introduction

Many processes in the petrochemical industry involve the blending of intermediate and final products. Large cost savings can be achieved by efficient blending schemes that satisfy the technical and regulatory specifications of products. For example, the economic and operability benefits from optimal crude-oil blend scheduling can reach multimillion dollars per year (Kelly and Mann, 2003).

One of the first mathematical programming models to represent the scheduling of blending operations is the pooling problem (Haverly, 1978). This problem seeks to find the optimal blend of materials available from a set of supply streams, while satisfying the demand of a set of products. The model enforces that the end products satisfy a specified minimum and maximum level for each specification. The objective is to minimize the total cost (or maximize the profit) of the operation. Several optimization models for the pooling problem have been reported in the literature. The  $p$ -formulation (Haverly, 1978), based on total flows and component compositions, is commonly used in chemical process industries. The  $q$ -formulation (Ben-Tal et al., 1994) uses variables based on the fraction that each input stream contributes to the total input to each pool, and does not explicitly use the pool specifications as variables. The  $pq$ -formulation (Tawarmalani and Sahinidis, 2002) is obtained by including valid redundant inequalities in the  $q$ -formulation. Tawarmalani and Sahinidis (2002) prove that the redundant constraints help to obtain a stronger polyhedral relaxation of the pooling problem. Lastly, Audet et al. (2004) propose a hybrid formulation by combining the  $p$  and  $q$  models to avoid additional bilinear terms that arise when generalized pooling problems are modeled using the  $q$ -formulation.

The multiperiod blending problem can be regarded as an extension of the pooling problem. In addition to the pooling problem restrictions, it considers inventory and time variations of supply and demand. The multiperiod blending problem can be formulated as a mixed-integer nonlinear programming (MINLP) problem (Kolodziej et al., 2013). Binary variables are required to model the movements of materials in and out of the tanks and to account for fixed costs. Even in the absence of binary variables, bilinear terms (which are necessary to model the mixing of various streams) make the problem nonconvex. Due to this highly combinatorial and nonconvex nature, the blend scheduling problem is very challenging. General purpose global optimization solvers fail to solve even small instances.

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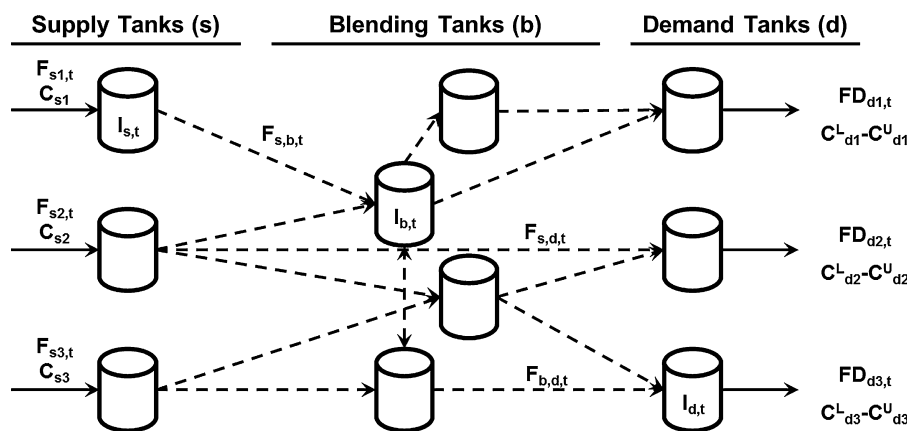


Fig. 1. Sketch of the multiperiod blending problem.

To the best of our knowledge, Foulds et al. (1992) were the first to propose a global optimization algorithm to solve a single-component pooling problem. They use McCormick envelopes (McCormick, 1976) to relax the bilinear terms. Androulakis et al. (1995) propose a convex quadratic NLP relaxation, known as  $\alpha$ BB underestimator. However, due to its generality, the NLP relaxation is weaker than its LP counterpart. Ben-Tal et al. (1994) and Adhya et al. (1999) present different Lagrangean relaxation approaches for developing lower bounds for the pooling problem. These bounds are tighter than standard LP relaxations used in global optimization algorithms.

The use of constraints that strengthen the linear relaxation of MINLP problems has been widely used in the literature. In particular, several authors use constraints that are redundant for the MINLP but cut off regions of the McCormick relaxation in MINLPs with bilinear terms. In the context of processing network problems, Quesada and Grossmann (1995) apply the reformulation-linearization technique (RLT) (Sherali and Adams, 1998), together with McCormick envelopes, to improve the relaxation of a bilinear program by creating redundant constraints. These authors combine concentration and flow based models in order to obtain a relaxed LP formulation that provides a valid and strong lower bound to the global optimum. Similar results are obtained by Tawarmalani and Sahinidis (2002) for the multicomponent pooling problem. The idea of using redundant constraints to strengthen the relaxation of the original problem is also used by Karupiah et al. (2008) in the context of water networks. These constraints correspond to total mass balance of contaminants and serve as deep cuts in the McCormick relaxation.

Piecewise MILP relaxations are an alternative relaxation of MINLPs that provide stronger bounds than traditional MILP relaxations. The first references to the use of piecewise MILP relaxation are by Bergamini et al. (2005) and Karupiah et al. (2008). Following this idea, Wicaksono and Karimi (2008) propose several novel formulations for piecewise MILP under and overestimators for bilinear programs. Gounaris et al. (2009) present a comprehensive computational comparison study of a collection of fifteen piecewise linear relaxations over a collection of benchmark pooling problems. Misener et al. (2011), building on the ideas from Vielma and Nemhauser (2011), introduce a formulation for the piecewise linear relaxation of bilinear functions with a logarithmic number of binary variables. Another alternative to piecewise relaxations are discretization techniques, such as multiparametric disaggregation (Kolodziej et al., 2013; Teles et al., 2013). The number of additional binary variables increases linearly with each increment in the precision of the discretization.

As an alternative to branch-and-bound solution procedures, Kolodziej et al. (2013) propose a heuristic as well as a rigorous two-stage MILP-NLP and MILP-MILP global optimization algorithms. Approximate and relaxed MILPs are obtained through the multiparametric disaggregation technique. Kesavan and Barton (2000) propose two approaches to generalize the outer approximation algorithm to separable nonconvex MINLP. Similarly, Bergamini et al. (2005), based on the work from Türkay and Grossmann (1996), present a deterministic algorithm based on logic-based outer approximation that can guarantee global optimality in the solution of the optimal synthesis of a process network problem.

Although the multiperiod blending problem arises in several applications, crude-oil blending is of great importance due to the potential increase in profit derived from optimal operation. In fact, crude-oil costs account for about 80% of the refinery turnover (Li et al., 2012). As a scheduling extension of the blending problem, crude-oil scheduling involves the unloading of crude marine vessels into storage tanks, followed by the transfer of crude from storage to charging tanks and finally, to the crude-oil distillation units (CDUs) (Lee et al., 1996; Shah, 1996). Lately, crude-oil scheduling models incorporate more quantity, quality, and logistics decisions related to real-life refinery operations, such as minimum run-length requirements, one-flow out of blender or sequence-dependent switchovers (Shah and Ierapetritou, 2011).

Several authors have proposed different algorithms relying on mixed-integer linear formulations to avoid solving the full nonconvex MINLP. These models can be seen as relaxations of the original MINLP. Méndez et al. (2006) present a novel MILP-based method where a very complex MINLP formulation is replaced by a sequential MILP approximation that can deal with non-linear gasoline properties and variable recipes for different product grades. Similarly, a two-stage MILP-NLP solution procedure is employed by Jia and Ierapetritou (2003) and Mouret et al. (2009), featuring in the first stage a relaxed MILP model without the bilinear blending constraints followed by the solution of the original MINLP after fixing all binary variables. The same two-stage algorithm is studied by Castro and Grossmann (2012) together with several global optimization methods. However, instead of dropping the bilinear constraints in the two-stage algorithm, they use multiparametric disaggregation to relax the bilinear terms. Moro and Pinto (2004) and Karupiah et al. (2008) tackle the problem with the augmented penalty version and a specialized version of the outer-approximation method, respectively. Reddy et al. (2004) propose an MILP relaxation combined with a rolling-horizon algorithm to eliminate the composition discrepancy. Finally, Li et al. (2012) use a spatial branch-and-bound global optimization algorithm, that at each node uses the MILP-NLP two-stage strategy previously mentioned, to solve the MINLP problems.

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