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Tentative engineering approach to scour around spherical bodies in random waves

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Abstract

An approach by which the scour depth around a spherical body and the self-burial depth of such a body in random waves can be derived is presented. Here the formulas for scour and self-burial depths of a spherical body by Truelsen et al. [Truelsen C, Sumer BM, Fredsøe J. Scour around spherical bodies and self-burial. ASCE J Waterway Port Coast Ocean Eng 2005;131(1):1–13] for regular waves are used. They are combined with describing the waves as a stationary Gaussian narrow-band random process to derive the scour and self-burial depths in random waves.

Keywords: Scour depth; Self-burial depth; Shear stress; Random waves; Stochastic approach

1. Introduction

The present work addresses the scour around a spherical body and the self-burial of such a body exposed to random waves. Examples of spherical bodies are stones in scour protection layers and sea mines on the seabed. Such bodies, which originally were installed, e.g., on a plane bed, may experience a range of seabed conditions, i.e. the bed may be flat or rippled; they may be surrounded by a scour hollow, and they may be self-buried. This is caused by the complicated three-dimensional flow generated by the interaction between the incoming flow velocity (e.g. the relative magnitude between waves and current), the geometry of the bed and the bed material, the ratio between the near-bed oscillatory fluid particle excursion amplitude and the characteristic dimension of the structure. Moreover, real waves are stochastic, making the problem more complex. The assessment of the scour around a spherical body is of interest in the design of, e.g., scour protection layers.

Further details on the general background and complexity of scour in the marine environment as well as reviews of the problems are given in, e.g., Whitehouse [2] and Sumer and Fredsøe [3]. To our knowledge, no studies are available in the open literature dealing with random wave scour around spherical bodies. The specifics related to scour around spherical bodies and self-burial of such bodies exposed to steady currents and regular waves are addressed in Truelsen et al. [1].

The purpose of this paper is to present a tentative engineering approach by which the scour depth around a spherical body and the self-burial depth of such a body in random waves can be derived. Here the empirical formulas based on data from laboratory tests for regular waves presented by Truelsen et al. [1] are used. The wave motion is assumed to be a stationary Gaussian narrow-band process, and the scour and self-burial depths are derived.

2. Scour and self-burial in regular waves

2.1. Scour

The scour around a spherical body in regular waves was investigated in laboratory tests by Truelsen et al. [1]. They obtained the following empirical formula for the equilibrium

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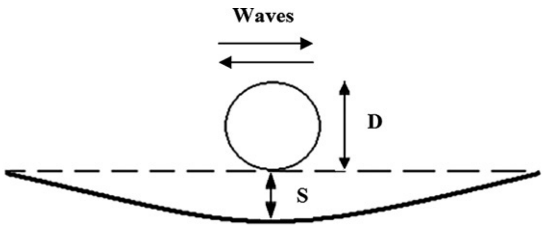

Fig. 1. Definition sketch of scour depth (S) around a spherical body (reproduced from Truelsen et al. [1]).

scour depth S around the spherical body with the diameter D (see Fig. 1)

$$\frac{S}{D} = 0.3\,\{1 - \exp\left[-0.3\ln(KC)\right]\} \quad \text{for } KC \geq 1. \tag{1}$$

The Keulegan–Carpenter number is defined as

$$KC = \frac{UT}{D} \tag{2}$$

where U is the undisturbed linear near-bed orbital velocity amplitude, and T is the wave period. Eq. (1) is valid for live-bed scour, for which $\theta > \theta_{\text{cr}}$, and θ is the undisturbed Shields parameter defined by

$$\theta = \frac{\tau_w}{\rho g(s-1)d_{50}} \tag{3}$$

where τ_w is the maximum bottom shear stress under the waves, ρ is the density of the fluid, g is the acceleration of gravity, $s = \rho_s/\rho$ is the sediment grain density to fluid density ratio, ρ_s is the sediment grain density, d_{50} is the median grain size diameter, and θ_{cr} is the critical value of the Shields parameter corresponding to the initiation of motion at the bed, i.e. $\theta_{\text{cr}} \approx 0.05$. One should note that the scour process attains its equilibrium stage through a transition period. Thus the approach is valid when it is assumed that the storm has lasted longer than the time scale of the scour. The major flow structures that cause scour around a spherical body placed on the seabed are the lee-wake vortices governed by KC; these vortices act as a mechanism to transport the eroded sediments away from the body during each half cycle of the motion. Further details on the timescale of the scour as well as on the mechanisms causing scour are given in Truelsen et al. [1].

The maximum bottom shear stress within a wave cycle is taken as

$$\frac{\tau_w}{\rho} = \frac{1}{2} f_w U^2 \tag{4}$$

where f_w is the wave friction factor, which here is taken as (Soulsby [4, Eq. 62a])

$$f_w = c\left(\frac{A}{z_0}\right)^{-d}; \quad c = 1.39,\ d = 0.52 \tag{5}$$

where $A = U/\omega$ is the near-bed orbital displacement amplitude, $\omega = 2\pi/T$ is the angular wave frequency, and $z_0 = d_{50}/12$ is the bed roughness. Eq. (5) is valid for rough turbulent flow under sinusoidal waves, and is obtained as best fit to data in the range $10 \lesssim A/z_0 \lesssim 10^5$. Moreover, Eq. (5)

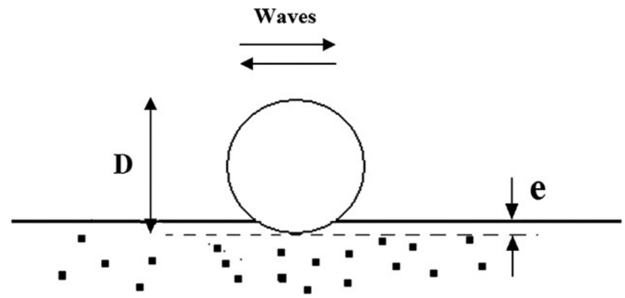

Fig. 2. Definition sketch of self-burial depth (e) around a spherical body (reproduced from Truelsen et al. [1]).

represents a compromise between simplicity and accuracy, and can be inverted to give an analytical distribution of the bed shear stress maxima.

One should notice that the KC number alternatively can be expressed as

$$KC = \frac{2\pi A}{D}. \tag{6}$$

Moreover, A is related to the linear wave height H by

$$A = \frac{H}{2\sinh kh} \tag{7}$$

where h is the water depth, and k is the wave number determined from the dispersion relationship $\omega^2 = gk\tanh kh$.

2.2. Self-burial

The self-burial of a spherical body in regular waves was also investigated in the laboratory tests by Truelsen et al. [1]. They obtained the following empirical formula for the equilibrium self-burial depth e of the spherical body with the diameter D (see Fig. 2)

$$\frac{e}{D} = 0.5\,\{1 - \exp\left[-q(KC - r)\right]\} \quad \text{for } KC \geq r \tag{8}$$

where q and r are coefficients given by the following values

$$(q, r) = (0.08, 1.4). \tag{9}$$

These results are valid for live-bed scour.

The main mechanism of self-burial of a spherical body is as follows: For a spherical body placed on the seabed the consequence of the developing scour is that the bearing area of the body is reduced causing an increased load on the soil; the bearing capacity of the soil will be exceeded, causing the soil to fail; the body will sink. The whole process will continue until the failure of the soil will stop, and consequently that the sinking ends. More details are given in Truelsen et al. [1].

3. Scour and self-burial in random waves

3.1. Outline of stochastic approach

For scour below pipelines and around vertical piles in random waves Sumer and Fredsøe [5,6] determined the properties of the random variables wave height H and wave