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1DV structure of turbulent wave boundary layers

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ABSTRACT

The 1DV structure of turbulent oscillatory boundary layers throughout the full relative roughness range $0 < r/A < 1$ and Reynolds numbers ($A^2\omega/\nu$) up to 6×10^6 is investigated through experimental data, A is the free stream semi excursion, r the bed roughness, ω the angular frequency and ν the kinematic viscosity. It is shown that smooth and rough flows alike can be scaled using the universal vertical scale $z_1 = \sqrt{2\nu/\omega + 0.0081rA}$, which becomes the Stokes' length $\sqrt{2\nu/\omega}$ for smooth beds ($r = 0$) and $0.09\sqrt{rA}$ for roughness dominated flows. Extraction of z_1 and hence r from experimental data provides a method for determining the effective hydraulic roughness of live sand-beds under waves in analogy with the log-fit method for determining the r from the zero intercept, $z_0 = r/30$ in steady flows. In fairly rough flows, e.g. over vortex ripples, the velocity structure is analogous to laminar flow and hence the eddy viscosity is constant with the value $\nu_t = 0.004\omega rA$, corresponding to $z_1 = 0.09\sqrt{rA} = \sqrt{2\nu_t/\omega}$. In the opposite limit of vanishing roughness and high Reynolds numbers, $z_1/A < 10^{-3}$, the velocity structure, which corresponds to von Karman's $\nu_t = ku_f z$, agrees reasonably with data. However, this asymptotic velocity structure deviates considerably from measurements through the naturally occurring r/A-range for sand beds. In the intermediate range $10^{-3} < r/A < 0.06$ the velocity structure does not seem to correspond to any real-valued, time invariant eddy viscosity. Comparison of the Kelvin function solution, which corresponds to $\nu_t = \kappa u_f z$, with smooth and almost-smooth, high Reynolds number measurements, shows that, the optimal zero intercepts are not the steady flow values $0.11\nu/u_f$ respectively $r/30$, but generally significantly larger. For low-roughness, high-Reynolds number oscillatory flows ($z_1/A < 0.002$), the magnitude of the velocity defect function universally follows $|D| \sim \exp[-(z/z_1)^{1/3}]$. This 1/3-power is interesting in that it has previously been shown to correspond maximum energy dissipation. In turbulent '2nd order Stokes flows' the vertical boundary layer scale of the second harmonic is smaller than that of the primary, but by a factor smaller than $2^{1/2}$, which applies in laminar flow. Density stratification in heavily sediment-laden sheet-flow makes the boundary layer thinner via the power p in $|D| \sim \exp[-z^p]$ taking greater values than in fixed bed experiments with similar z_1/A.

1. Introduction

The next generation of operational coastal hydrodynamic and morphodynamic models requires an effective 1DV description of the wave boundary layer and the associated bed shear stresses and eddy-viscosities.

The following presents such a description which is compared with all available experimental data, covering the full range from fairly rough flows $r/A \approx 0.2$, which occur over sharp crested vortex ripples, down to the smooth, turbulent asymptote.

For operational models, which are 1DV, as well as for analytical work, it is natural to work with a simple harmonic boundary layer structure quantified by its time-invariant velocity defect function $D(z)$, or flows which can be considered as super-positions of simple harmonic components $u_j(z,t)$, each with their own defect function $D_j(z)$. Hence, we start with updating experimental knowledge about D. The main advancements being universal smooth/rough scaling and parameter limits between: the laminar-like rough-turbulent (big r/A, cf. Nielsen, 1992, Fig. 1.2.17), the intermediate, and the quasi-steady flow structures applicable for low relative roughness.

Data from the full relative roughness range is found to support the description $|D| \sim \exp[-(z/z_1)^p]$ with the vertical scale being $z_1 = \sqrt{2\nu/\omega + 0.0081rA}$, i.e., a geometrical combination of the viscous Stokes length and the fully rough vertical scale $0.09\sqrt{rA}$. The latter expression was found by Nielsen (1992, page 46) to be supported by all fixed bed experiments with known roughness performed up to that time. The role of the power p is to stretch, respectively compress, the boundary layer structure vertically compared with the laminar-like solution, which has $p = 1$. Thus, for low relative roughness and/or high Reynolds numbers where $z_1 << A$, the fixed bed data indicates $p < 1$ with the lower limit $p \to 1/3$ for $z_1/A \to 0$.

While fixed bed data are thus restricted to $1/3 < p < 1$, the structure of densely sediment-laden sheet-flows is found to be compressed, most

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likely due to density stratification, with p taking values in the range $1 < p < 1.4$.

The question of similar or otherwise structure for the primary versus higher harmonic velocity components is addressed through analysis of a sheet-flow dataset with 2nd order Stokes forcing. This dataset is also used as an example, where the velocity defect analysis is used to determine the hydraulic roughness of a live sand bed.

2. The velocity defect function for simple harmonic flow

For a 1DV scenario, i.e., assuming $\frac{\partial}{\partial x} \equiv \frac{\partial}{\partial y} \equiv 0$ the equation of motion is

$$\rho \frac{\partial u}{\partial t} = \frac{\partial \tau}{\partial z} - \frac{\partial p}{\partial x} \tag{1}$$

where ρ is the density, p is the pressure and τ is the shear stress. Aiming for simplification, we introduce the complex velocity defect function defined by

$$u(z,t) = [1-D]u_\delta(t). \tag{2}$$

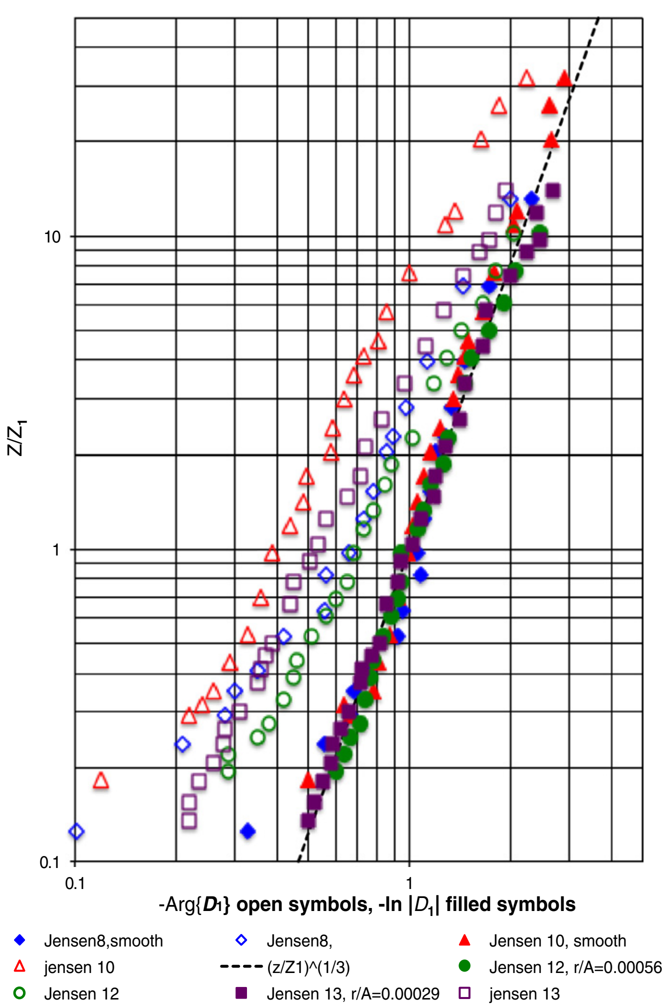

Fig. 1. Experimental values of ln|D| and Arg{D} based on the primary harmonic components from the smooth and almost-smooth, high Reynolds number (1.6×10^6 and 6.0×10^6) measurements of Jensen (1989). The experiment parameters are summarized in Table 1.

The free stream velocity above the boundary layer is $u_\delta(t)$, and the pressure through the boundary layer is assumed hydrostatic so that, at all levels:

$$\frac{\partial p}{\partial x} = -\rho \frac{\partial u_\delta}{\partial t}. \tag{3}$$

With the eddy-viscosity defined by $\tau = \nu_t \frac{\partial u}{\partial z}$ the equation of motion then becomes

$$\frac{\partial (D u_\delta)}{\partial t} = \frac{\partial}{\partial z}\left(\nu_t \frac{\partial D}{\partial z}\right) u_\delta \tag{4}$$

and for simple harmonic free stream conditions, with velocities taken as the real part of complex functions, i.e., $u_\delta(t) = U_\delta Re\{e^{i\omega t}\}$, one gets the following differential equation for the defect function:

$$i\omega D + \frac{\partial D}{\partial t} = \frac{\partial}{\partial z}\left(\nu_t \frac{\partial D}{\partial z}\right). \tag{5}$$

If ν_t is not a function of time, D is a function of z only. In particular, if ν_t is a constant, similarly to the laminar case, the solution is similar to the laminar solution:

$$D = e^{-(1+i)\frac{z}{\sqrt{2\nu/\omega}}} = e^{-\frac{z}{\sqrt{2\nu/\omega}}}\left(\cos\frac{z}{\sqrt{2\nu/\omega}} - i\sin\frac{z}{\sqrt{2\nu/\omega}}\right) \tag{6}$$

The geometrical properties of $D(z)$ in the complex plane are comprehensively illustrated in Nielsen (1985,1992) pp. 20–22.

2.1. $D(z)$ and ν_t for fairly rough beds

For relatively rough beds $r/A > 0.06$ experiments show that the boundary layer structure is analogous to that of laminar flow. That is, the velocity defect function has the form

$$D = e^{-(1+i)\frac{z}{\sqrt{2\nu_t/\omega}}} = e^{-(1+i)\frac{z}{0.09\sqrt{rA}}} \tag{7}$$

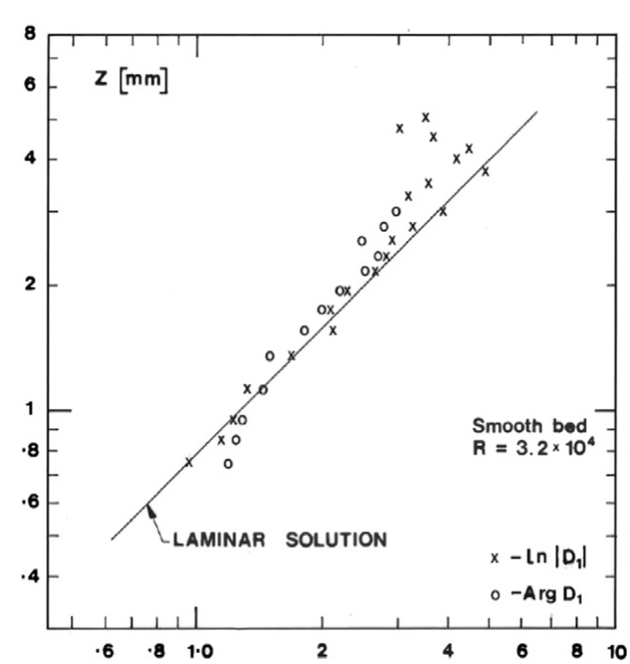

Fig. 2. $-$Arg{D} and $-$ln|D| from a 'moderately turbulent' smooth flow plotted as function of z and compared with the laminar solution. Data from van Doorn and Godefroy (1978).