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Internal generation of damped waves in linear shallow water equations

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ABSTRACT

Techniques to generate damped waves in linear shallow water equations are developed. Delta-shaped source functions are derived for four different cases with the source function and damping coefficient either in the continuity or momentum equation. Source terms are also found by relating the source function method to the source term addition method. For the transport of the source term in damped waves, we define complex valued source-term velocity which is different from the real valued energy velocity. We verify the present wave generation techniques with the source-term velocity by simulating damped waves on a horizontal bottom both in the one- and two-dimensional domains, and also damped waves shoaling and refracting on a plane slope in the two-dimensional domain.

1. Introduction

Coastal engineers use a finite computational domain to understand wave transformations. Some waves may propagate from the interior to the outside boundary and should not be returned into the domain. Thus, the engineers need to specify open boundary condition that the outgoing waves propagate freely without any numerical problem. One such technique is to generate waves internally inside the domain at a wave generation region and put a sponge layer at the outside boundary. There are two methods of internal generation. One is to add a source term such as water surface elevation or particle velocity at the wave source region at each time step during the computation procedure. Another is to put a source function in the continuity or momentum equation of the governing equations. Kim et al. (2007) found that the source term addition method is equivalent to the source function method. Larsen and Dancy (1983) first used the source term addition method for Peregrine's (1967) Boussinesq equations. They found that the source term propagates with the phase velocity explaining that water mass propagates with the phase velocity. Later, Lee and Suh (1998) found that the source term propagates with the energy velocity instead of the phase velocity. The energy velocity is a group velocity that depends on the wave equations. For example, for mild-slope equations of Copeland (1985) and Radder and Dingemans (1985), the energy velocity is the phase velocity and the group velocity, respectively. Using the energy velocity for the transport of the source term, Lee et al. (2001) succeeded in generating waves internally for extended Boussinesq equations of Nwogu (1993). They explained that, for Peregrine's Boussinesq equations, the energy velocity is close to the phase velocity in shallow water and thus Larsen and Dancy could generate waves using the phase velocity. The concept of energy propagation in the internal generation of waves was theoretically explained by Schäffer and Sørensen (2006) who investigated wave generation mechanism in a grid system of time and space and also by Kim et al. (2007) who used the Green's second identity. Wei et al. (1999) first used the source function method by putting the source functions in either the continuity or momentum equation of the Boussinesq equations. They used the Gaussian-shaped source function in a source region instead of the previously used delta-shaped source function which was used in a source line. Also, Kim et al. (2006) developed source functions in a source region for several types of the mild-slope equations. To generate multi-directional waves, at least two wave generation regions, i.e., one in the main direction and the other in the orthogonal direction, are needed. Then, for obliquely incident waves, the wave diffraction problem would occur near the points where the two regions cross over because target wave energy is over- or under-specified at these points. To overcome such a diffraction problem, Lee and Yoon (2007) suggested using an arc-shaped wave generation line connecting the two orthogonal lines. Further, Kim and Lee (2013) developed an arc-shaped wave generation band yielding errors less than the arc-shaped line. The aforementioned wave equations for internal generation were of the hyperbolic type. The internal wave generation was also made in the elliptic mild-slope equation of Berkhoff (1972) by Bellotti et al. (2003). The method of internal wave generation was also used in the three-dimensional Navier–Stokes equations by several researchers (Choi and Yoon, 2009; Ha et al., 2013; Kawasaki, 1999; Lin and Liu, 1999). Another method of specifying open boundary condition is putting a relaxation zone where target waves are generated properly and waves reflected from the interior are absorbed smoothly (Madsen et al., 2003; Mayer

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et al., 1998). The method was found to be successful in generating obliquely incident waves with the two-dimensional Boussinesq equations (Jamois et al., 2006) and also waves with the three-dimensional Navier–Stokes equations (Jacobsen et al., 2012).

To date, coastal engineers have developed techniques of generating waves internally for un-damped waves. In actual sea, wave energy may dissipate due to turbulence in the surf zone or bottom friction in shallow water, etc. One way to simulate energy dissipation is to put the damping coefficient in the momentum or continuity equation, and the magnitude of damping coefficient is determined by fitting the magnitude to the value determined by a physical formula (Dalrymple et al., 1984; Reid and Kajiura, 1957). In this study, we develop a technique of internal generation of damped waves in linear shallow water equations. The source function for wave generation is considered to be delta shaped. In Section 2, we use the geometric optics approach to get a dispersion relation, a relation among the damping coefficient and the wave number, and the energy velocity. In Section 3, we develop delta-shaped source functions for four different cases with the source function and damping coefficient in the continuity and momentum equations. We also get source terms using the fractional step splitting method and define a so-called source-term velocity which is different from the energy velocity for the transport of the source term for damped waves. In Section 4, we verify the present wave generation techniques by simulating damped waves on a horizontal bottom in both the one- and two-dimensional domains. We also conduct damped waves shoaling and refracting on a plane slope in the two-dimensional domain. In Section 5, we summarize the present study.

2. Geometric optics method

The linear shallow water equations in the one-dimensional domain can be described as

$$\frac{\partial \eta}{\partial t}+\frac{\partial (hu)}{\partial x}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+g\frac{\partial \eta}{\partial x}=0 \tag{2}$$

where η is the water surface elevation, u is the horizontal particle velocity, h is the still water depth, and g is the gravitational acceleration. The source function may be included in the continuity or momentum equation. Also, the damping term may be included in the continuity or momentum equation. Thus, there may be four different cases with source function and damping term in the equations, that is, case (1-1): source and damping in the continuity equation, case (1-2): source in the continuity equation and damping in the momentum equation, case (2-1): source in the momentum equation and damping in the continuity equation, and case (2-2): source and damping in the momentum equation. In this section we use the geometric optics approach to get the dispersion relation, the relation among the damping coefficient and the wave number, and the energy velocity in the linear shallow water equations with a damping coefficient.

The continuity equation with the damping term $D_c\eta$ as in cases (1-1) and (2-1) can be described as

$$\frac{\partial \eta}{\partial t}+\frac{\partial (hu)}{\partial x}+D_c\eta=0. \tag{3}$$

And, the momentum equation with the damping term $D_m u$ as in cases (1-2) and (2-2) can be described as

$$\frac{\partial u}{\partial t}+g\frac{\partial \eta}{\partial x}+D_m u=0. \tag{4}$$

In Eqs. (3) and (4), D_c and D_m are the damping coefficients in the continuity and momentum equation, respectively. First, we consider the case with the damping coefficient in the momentum equation such as cases (1-2) and (2-2). For the constant damping coefficient, differentiating Eq. (1) in time and using Eqs. (1) and (4) yield the following equation

$$\frac{\partial^2 \eta}{\partial t^2}-g\frac{\partial}{\partial x}\left(h\frac{\partial \eta}{\partial x}\right)+D_m\frac{\partial \eta}{\partial t}=0. \tag{5}$$

The water surface elevation can be expressed as

$$\eta=a_0e^{i(kx-\omega t)}=ae^{i(k_r x-\omega t)} \tag{6}$$

where $a=a_0e^{-k_i x}$, ω is the angular frequency, $k(=k_r+i\,k_i)$ is the complex wave number, k_r is the real part related to wave phase, and k_i is the imaginary part related to the decay of wave amplitude. Substituting Eq. (6) into Eq. (5) gives, in real part, the dispersion relation given by

$$C_p=\frac{\omega}{k_r}=\sqrt{gh\left[1-\left(\frac{k_i}{k_r}\right)^2\right]} \tag{7}$$

where C_p is the phase velocity. And, in the imaginary part, we get the relation between the damping coefficient D_m and the complex wave number k given by

$$D_m=2\frac{gh}{\omega}k_r k_i=2k_i C_e \tag{8}$$

and also the energy transport equation given by

$$\frac{\partial}{\partial x}\left(\frac{gh}{\omega}k_r a_0^2\right)=\frac{\partial}{\partial x}\left(C_e a_0^2\right)=0. \tag{9}$$

In Eqs. (8) and (9), C_e is the energy velocity given by

$$C_e=\sqrt{\frac{gh}{1-\left(\frac{k_i}{k_r}\right)^2}}=\frac{C_p}{1-\left(\frac{k_i}{k_r}\right)^2}. \tag{10}$$

From Eqs. (7) and (8), we can get the wave number ratio given by

$$\frac{k_i}{k_r}=\frac{\sqrt{1+\left(\frac{D_m}{\omega}\right)^2}-1}{\frac{D_m}{\omega}}. \tag{11}$$

Substitution of Eq. (11) into Eq. (7) yields the solution for k_r and using it in Eq. (11) yields the solution for k_i.

We also use the geometric optics approach for the damping coefficient in the continuity equation as in cases (1-1) and (2-1), and get the same relations given by Eqs. (7)–(10).

3. Techniques for the internal generation of damped waves

The derivation procedure to get the source function is similar among the four different cases with the source function and damping coefficient. In this study, we show a detailed derivation procedure for cases (1-2) and (2-2) and show the final form of the developed source function for cases (1-1) and (2-1). We also find source terms that are used in the source term addition method (Kim et al., 2007).

3.1. Development of source functions

For case (1-2) in the horizontally one-dimensional domain, the continuity equation with the source function S_c is given by

$$\frac{\partial \eta}{\partial t}+\frac{\partial}{\partial x}(hu)=S_c \tag{12}$$