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# Absorbing–generating seaward boundary conditions for fully-coupled hydro-morphodynamical solvers

Giorgio Incelli *, Riccardo Briganti, Nicholas Dodd

*University of Nottingham, University Park, Nottingham NG7 2RD, UK*

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ABSTRACT

This paper presents a new technique to compute open boundary conditions for fully-coupled hydro-morphodynamical numerical solvers based on the Non-Linear Shallow Water and the Exner equations. These conditions allow the generation of incident signals and the absorption of reflected ones, taking into account the bed evolution at the boundary. They use the approximations for linear waves in shallow water and are based on the solution of the Riemann Equations.

The proposed technique is implemented in the fully-coupled hydro-morphodynamical numerical model of Briganti et al. (2012a).

Firstly, the generation and absorption of single monochromatic waves are studied to quantify the error after the reflected wave exited the domain. In all cases the error is always small, giving evidence of the effectiveness of the new seaward boundary conditions.

Furthermore, the propagation and reflection of a monochromatic wave train over a mobile bed are considered. Both flow evolution and bed change are not affected by spurious oscillations when long sequences of waves are tested. Additionally, a very low mobility bed is considered to simulate a 'virtually fixed' bed and new boundary condition results consistently converge to those for the hydrodynamic only case.

Finally, the reflection of a uniform bore over a mobile bed is studied. For this case the Rankine–Hugoniot conditions provide an analytical solution. It is apparent that the adopted linear approximations produce errors in the velocity estimates. Nevertheless, the conditions perform reasonably well even in this demanding non-linear case.


## 1. Introduction

In the nearshore region the flow strongly interacts with the bottom sediments, shaping the bed. In the swash zone, in particular, the oscillatory flow and the bed evolve with the same time scale (Kelly and Dodd, 2010). This consideration has led to the development of hydro-morphodynamical numerical models that are able to solve the equations for the flow and bed evolution simultaneously, i.e., enabling full coupling.

In coastal research, fully-coupled solvers are typically applied to a system of equations comprising the Non-Linear Shallow Water Equations (henceforth NLSWEs) and the Exner equation.

Given the limitations of the NLSWEs, these models are frequently forced to locate the seaward boundary at depths where the sediment is still active. Therefore boundary conditions should be able to prescribe the incoming flow and sediment while allowing the outgoing quantities to exit the computational domain. At present, to the best of authors' knowledge, such conditions for fully-coupled models have not been formulated. In morphodynamic numerical models where frequency dispersion is included, it is common practice to locate the seaward boundary where depth of closure is reached, typically at depths for which the shallow water approximation is no longer appropriate. This occurs with well established solvers, such as Genesis (Hanson and Kraus, 1989) and XBeach (Roelvink et al., 2009), which include submodels for wave propagation from deep to shallow water.

Fully-coupled models were first used in Hudson and Sweby (2003) and Hudson et al. (2005), who developed a finite volume solver and tested it on a dune migration problem. Later, finite volumes were employed by Dodd et al. (2008) to study the formation of beach cusps. More recently Kelly and Dodd (2010), Briganti et al. (2012b), Zhu et al. (2012) and Zhu and Dodd (2013) used different solvers to simulate bore-driven swash flows. All these works only considered bed load transport. Furthermore, in these works the absorbing-generating seaward boundary conditions have not been used because a single swash event was studied and the seaward boundary was located far from the region of interest so that any reflected perturbation would not interfere with the event considered.

Absorbing–generating conditions are available for the hydrodynamic equations (e.g., Kobayashi et al., 1987; van Dongeren and Svendsen, 1997) and are usually based on the knowledge of the Riemann invariants. They were used also in morphodynamic simulations following a

* Corresponding author.
*E-mail addresses:* evxgi1@nottingham.ac.uk (G. Incelli), riccardo.briganti@nottingham.ac.uk (R. Briganti), nicholas.dodd@nottingham.ac.uk (N. Dodd).

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simplified coupled approach (Dodd et al., 2008). Specifically, closure of the problem was achieved by priming the updated bed value at the seaward boundary with that of the first inner cell at the previous time step, i.e., imposing a horizontal bottom therein.

Savary and Zech (2007) presented a fully-coupled characteristic-based approach for boundaries in fluvial environment. This implies that oscillatory flows are not considered. Besides, this technique is suitable for a two fluid layer model, following the Fraccarollo and Capart (2002) approach, in which a second fluid layer where water and sediment are mixed is considered.

The present study proposes new fully-coupled absorbing-generating seaward boundary conditions for oscillatory flows. These are based on the solution of the Riemann Equations, following previous work by Kelly and Dodd (2009) and Zhu and Dodd (2013). They include approximations for linear waves in shallow water, so that only the incoming water surface perturbation is needed to determine updated quantities at the seaward boundary.

This paper is organised in four sections. Section 1 shows the governing equations, while in Section 2 the new fully-coupled seaward boundary conditions are explained. Section 3 presents the validation tests and Section 5 summarises the achieved results.

## 2. Governing equations

### 2.1. Definitions

The model used in this study solves the one-dimensional NLSWEs coupled with the Exner sediment continuity equation for the bed evolution.

The above-mentioned equations form a system of conservation laws with source terms:

$$\begin{bmatrix} h \\ uh \\ z_b \end{bmatrix}_t + \begin{bmatrix} uh \\ hu^2 + \frac{1}{2}gh^2 \\ \xi q_s \end{bmatrix}_x = \begin{bmatrix} 0 \\ -gh\frac{\partial z_b}{\partial x} \\ 0 \end{bmatrix}, \quad (1)$$

where $x$ and $t$ are the independent variables (space and time respectively); $g$ is the gravitational acceleration; $h$, $u$ and $z_b$ are the dependent variables, namely the local water depth, the depth-averaged horizontal velocity and the bed level in the order. Fig. 1 shows the variables of the hydro-morphodynamic system.

Additionally, $\xi = 1/(1-p_b)$, where $p_b$ is the bed porosity, and $q_s$ is the instantaneous bed load sediment transport, for which the well-known Grass formula is used

$$q_s = Au^3, \quad (2)$$

with $A$ being the sediment mobility parameter. Note that bed load sediment transport is appropriate for medium-coarse sand environments.

System (1) is solved using the TVD-MacCormack scheme (hereinafter TVD-MCC) from Briganti et al. (2012a), which is restated in Appendix A.

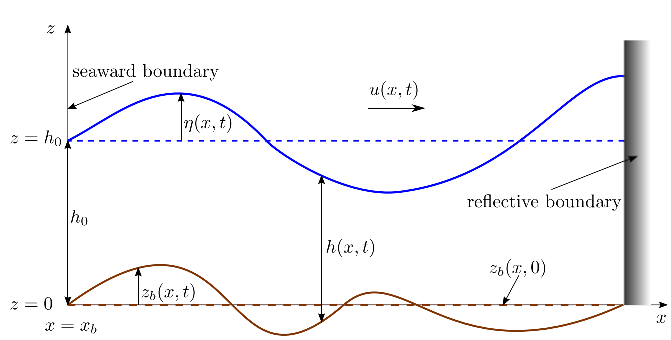


**Fig. 1.** Sketch of variables for a generic mobile bed problem with a seaward (left) boundary and a reflective (right) one.

## 3. Fully-coupled seaward boundary conditions

### 3.1. Formulation of conditions

In this section, the new fully-coupled absorbing–generating seaward boundary conditions, named a Riemann Equation Boundary Conditions (henceforth REBCs), are presented.

The NLSWEs-Exner System (1) is rewritten in primitive form:

$$\begin{bmatrix} h \\ u \\ z_b \end{bmatrix}_t + \begin{bmatrix} hu \\ \frac{1}{2}u^2 + g(h+z_b) \\ \xi q_s \end{bmatrix}_x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (3)$$

It is assumed that the flow at the seaward boundary is subcritical and that approximations for linear waves in shallow water can be used therein. These are essentially the same assumptions made in Kobayashi et al. (1987).

The seaward boundary is located at $x = x_b$ and the incoming waves propagate from the left to the right of Fig. 1. Thus $h(x_b, t)$ can be computed from

$$h(x_b,t) = h_0 + z_b(x_b,0) - z_b(x_b,t) + \eta_i(x_b,t) + \eta_r(x_b,t), \quad (4)$$

where $h_0$ is the still water depth at the seaward boundary and $\eta$ is the perturbation from the initial water surface (i.e., $h_0 + z_b(x_b, 0)$) due to the incident ($\eta_i$) and reflected ($\eta_r$) waves respectively. In particular, $\eta_i(x_b, t)$ is known while the initial reference bed level is set $z_b(x_b, 0) = 0$ (see Fig. 1) and therefore omitted in the following.

The water velocity at the seaward boundary $u(x_b, t)$ can be computed as

$$u(x_b,t) = u_i(x_b,t) + u_r(x_b,t), \quad (5)$$

with

$$u_i(x_b,t) \approx \eta_i(x_b,t)\sqrt{\frac{g}{h_0 - z_b(x_b,t)}}, \quad (6)$$

$$u_r(x_b,t) \approx -\eta_r(x_b,t)\sqrt{\frac{g}{h_0 - z_b(x_b,t)}}. \quad (7)$$

$u_i(x_b, t)$ is a function of $\eta_i(x_b, t)$, which is known, and $z_b(x_b, t)$. $u_r(x_b, t)$ is related to $z_b(x_b, t)$ and $\eta_r(x_b, t)$ through Eq. (7). The new technique makes use of two of the three Riemann Equations associated to System (3) at the seaward boundary to determine two unknowns, $z_b(x_b, t)$ and $\eta_r(x_b, t)$, with the help of Eqs. (4), (6) and (7).

The generic Riemann Equation (see Zhu, 2012 for the derivation) is written as:

$$\Re^k = \frac{Dz_b}{Dt} + \frac{\lambda_k}{\lambda_k - u}\frac{Dh}{Dt} + \frac{\lambda_k}{g}\frac{Du}{Dt} = 0, \quad (8)$$

where $\frac{D}{Dt}$ indicates the total (material) derivative and $\lambda_k$ ($k = 1, 2, 3$) are the eigenvalues of the Jacobian matrix associated to System (3). Such eigenvalues are computed numerically (e.g., Kelly and Dodd, 2009) as no analytical expression is available for the morphodynamic problem.

At the seaward boundary, $\lambda_1$ is positive, $\lambda_2$ is negative, while:

$$\lambda_3 > 0 \text{ if } u > 0,$$

$$\lambda_3 < 0 \text{ if } u < 0.$$