Coastal Engineering 93 (2014) 55–70

Contents lists available at ScienceDirect

Coastal Engineering

journal homepage: www.elsevier.com/locate/coastaleng

High order Hamiltonian water wave models with wave-breaking mechanism

R. Kurnia [a,*], E. van Groesen [a,b]

[a] Applied Mathematics, University of Twente, The Netherlands
[b] LabMath-Indonesia, Bandung, Indonesia

ARTICLE INFO

Article history:
Received 11 September 2013
Received in revised form 14 July 2014
Accepted 6 August 2014
Available online 6 September 2014

Keywords:
Variational modelling
Wave-breaking
Eddy-viscosity
Kinematic criterion
AB-equation

ABSTRACT

Based on the Hamiltonian formulation of water waves, using Hamiltonian consistent modelling methods, we derive higher order Hamiltonian equations by Taylor expansions of the potential and the vertical velocity around the still water level. The polynomial expansion in wave height is mixed with pseudo-differential operators that preserve the exact dispersion relation. The consistent approximate equations have inherited the Hamiltonian structure and give exact conservation of the approximate energy. In order to deal with breaking waves, we extend the eddy-viscosity model of Kennedy et al. (2000) to be applicable for fully dispersive equations. As breaking trigger mechanism we use a kinematic criterion based on the quotient of horizontal fluid velocity at the crest and the crest speed. The performance is illustrated by comparing simulations with experimental data for an irregular breaking wave with a peak period of 12 s above deep water and for a bathymetry induced periodic wave plunging breaker over a trapezoidal bar. The comparisons show that the higher order models perform quite well; the extension with the breaking wave mechanism improves the simulations significantly.

1. Introduction

Accurate simulations of waves in deep water and in the coastal zone are important for various offshore activities and environmental issues. Efficiency and safety in design, installation and operations are most important for offshore wind farms, oil and gas platforms, ship and harbour design and for sustainable coastal management. Accurate wave models are needed to predict and describe waves also in extreme cases. This paper aims to contribute in presenting good and efficient models that are capable to describe rough waves and breaking waves.

The paper starts with the derivation of higher order Boussinesq-type models with exact dispersion by using the basic variational formulation of incompressible, irrotational surface waves with free surface under the influence of gravity. Nonlinear gravity waves have been the object study in many theoretical, numerical and experimental investigations. Boussinesq (1872) simplified the Euler equations for irrotational, incompressible fluid by approximating the Laplace equation for the interior fluid potential to obtain equations in horizontal quantities only by approximating the depth dependence. This leads to bi-directional and dispersive dynamic equations for the surface elevation and a fluid velocity. Korteweg and de Vries (1895) (KdV) derived special solutions, the soliton and periodic equivalents, of a simplified Boussinesq equation for which the velocity variable is related to the surface elevation in such a way that the two dynamic equations lead to one unidirectional dynamic equation.

Zakharov (1968) formulated the basic Hamiltonian formulation of water waves on the surface of an infinitely deep fluid. The Hamiltonian in this formulation contains the kinetic energy, which is the Dirichlet integral of the fluid potential, that has to be expressed for given surface elevation in the other canonical variable which is the fluid potential at the surface. Craig and Sulem (1993) approximated Zakharov's formulation up to fifth order accuracy by a Taylor expansion of the Dirichlet-to-Neumann operator that maps the fluid potential at the fluid surface to the normal derivative of fluid potential at the surface. Recent KdV-type of models for waves above finite or infinite depth, called AB equations, have been developed by van Groesen and Andonowati (2007). Using a second order Taylor expansion for the surface potential and for the vertical velocity around the still water level, leads to an approximation with exact dispersion in first and second order terms.

In this paper we present a simple derivation of higher order Hamiltonian equations for bi-directional waves. Just as in van Groesen and Andonowati (2007) an expansion around the still water level will be used to approximate the exact kinetic energy to any desired order in the wave amplitude, keeping the exact dispersion properties. To that end, the Taylor approximation of the normal velocity in the still water potential is expressed in the desired potential at the free surface after inverting the expansion of the free surface potential in the still water potential. The dynamic equations are then obtained by taking variations of the Hamiltonian which is explicitly expressed in the canonical variables. By invoking a uni-directionalisation assumption, higher order

* Corresponding author.
E-mail address: r.kurnia@utwente.nl (R. Kurnia).

http://dx.doi.org/10.1016/j.coastaleng.2014.08.002
0378-3839/

KdV equations can be obtained as extensions of the AB equations; therefore the higher order Hamiltonian equations will be called AB Hamiltonian Systems (ABHS).

Following the basic ideas used for other type of Boussinesq equations, we will extend the ABHS equations with a mechanism to deal with breaking waves. To that end, a trigger mechanism for the initiation of the wave breaking, and an energy dissipation mechanism have to be chosen.

Three dominant types of dissipation models in the current literature are the surface roller model (Schäffer et al. (1993); Madsen et al. (1997)), the vorticity model (Svendsen et al. (1996); Veeramony and Svendsen (2000)) and the eddy viscosity model (Heitner and Housner (1970); Zelt (1991); Kennedy et al. (2000)). For the initiation of the breaking, different methods have been described: the trigger mechanism based on the slope angle variation (Schäffer et al. (1993)), based on the normal speed of the free surface elevation exceeding some threshold value (Kennedy et al. (2000)), Relative Trough Froude Number (RTFN) (Okamoto and Basco (2006)) and recently the Breaking Celerity Index method that couples the criterion proposed by Kennedy and the RTFN (D'Alessandro and Tomasicchio (2008)).

In this paper, we will implement for the ABHS models an extension of the eddy viscosity breaking model of Kennedy et al. (2000). The extension makes it possible to deal with fully dispersive waves and will be applicable not only in shallow water but also in deep water. Besides that, we will investigate two variants for the viscosity coefficient. In one variant the decay is determined by the normal velocity as in Kennedy et al. (2000), while in a second variant the decay is determined by the tangential velocity; both variants will lead to almost similar results.

As trigger mechanism, we use the kinematic breaking criterion that the wave will break when the horizontal particle speed exceeds (a fraction of) the crest speed. The crest speed will be determined by an explicit expression of the local wavenumber as suggested by Stansell and MacFarlane (2002) by applying the spatial Hilbert transform; this mechanism will shown to be quite robust and applicable for all water depths.

The organisation of the paper is as follows. In Section 2 we present the variational description of surface waves and the consistent approximation of the ABHS equations up to fourth order. Section 3 deals with the extension to wave breaking with the eddy viscosity model and the kinematic breaking criterion. The numerical implementation with a pseudo-spectral code is briefly described in Section 4. In Section 5 we show results of simulations and compare these with accurate data. Experiments of deep water, irregular, breaking waves are available from the hydrodynamic laboratory MARIN (Maritime Research Institute Netherlands). Bathymetry induced breaking is compared with experiments of periodic long waves plunging breaking over a bar by Beji and Battjes (1993). Conclusions and remarks will finish the paper.

2. Variational wave description

In Section 2.1 we start with the description of the Hamiltonian formulation for surface water waves. In the expression of the kinetic energy functional appears the vertical velocity implicitly defined as operator linear in the surface potential and nonlinear in the elevation. In Section 2.2 we approximate the kinetic energy by using Taylor expansion with the potential at the still water level as intermediate variable. The approximation of the kinetic energy in second to fifth order leads to approximations of the dynamic equations with first to fourth order accuracy. We will verify that the Hamiltonian with the approximate kinetic energy leads to the same results as an approximation of the exact equations; as one consequence of this Hamiltonian consistent modelling, exact conservation of the approximate energy is guaranteed. In Section 2.3 we describe a Hybrid Spatial Spectral method to deal with varying bottom in the application in Section 5.2

2.1. Hamiltonian formulation

Zakharov (1968), and later independently Broer (1974), showed that waves in one horizontal direction x on the surface of an incompressible, inviscid fluid under the influence of gravity can be described by a set of Hamilton equations for the surface elevation $\eta(x, t)$ and the surface fluid potential $\phi(x, t)$ as canonical variables. Miles (1977) showed that this could have been derived from a variational pressure principle as formulated by Luke (1967) that could easily be rewritten as an action functional

$$\int\left[\int \phi\partial_t\eta\, dx - \mathcal{H}(\phi,\eta)\right]dt \tag{1}$$

for which the critical points, just as in Classical Mechanics, satisfy the Hamilton equations:

$$\partial_t\eta = \delta_\phi\mathcal{H}(\phi,\eta)$$
$$\partial_t\phi = -\delta_\eta\mathcal{H}(\phi,\eta).$$

Here we use the notation δ_ϕ and δ_η to denote the variational derivative with respect to ϕ and η respectively.

The Hamiltonian is the total energy, the sum of potential and kinetic energy:

$$\mathcal{H}(\phi,\eta) = \frac{1}{2}\int g\eta^2\, dx + \mathcal{K}(\phi,\eta)$$

where the kinetic energy is formally given for finite and infinite depth, by

$$\mathcal{K}(\eta,\phi) = \frac{1}{2}\int\int |\nabla\Phi|^2\, dx\, dz.$$

Here Φ is the fluid potential that satisfies the Laplace equation in the interior fluid domain (representing the incompressibility condition for irrotational fluid motion), the surface condition $\Phi = \phi$ at $z = \eta$ and the impermeable bottom boundary condition. By applying Green's theorem, the kinetic energy can be expressed as

$$\mathcal{K}(\eta,\phi) = \frac{1}{2}\int \phi\partial_{\mathbf{n}}\Phi\, dx. \tag{2}$$

With W the vertical velocity $W = \Phi_z(x,\eta)$, here $\partial_{\mathbf{n}}\Phi = W - \eta_x\Phi_x$ at $z = \eta$ is the normal velocity at the surface, the Dirichlet-to-Neumann operator. Since $\phi_x = \Phi_x(x,\eta) + \eta_x W$, we get $\partial_{\mathbf{n}}\Phi = W(1+\eta_x^2) - \eta_x\phi_x$ and the kinetic energy can be rewritten as

$$\mathcal{K}(\phi,\eta) = \frac{1}{2}\int \phi\left\{W\left(1+\eta_x^2\right) - \eta_x\phi_x\right\}dx. \tag{3}$$

It holds that $\delta_\phi\mathcal{K} = \partial_{\mathbf{n}}\Phi$, as shown by Zakharov (1968), and we get for the first Hamilton equation $\partial_t\eta = \delta_\phi\mathcal{H}(\phi,\eta) = W(1+\eta_x^2) - \phi_x\eta_x$. This is the kinematic surface condition, the continuity equation.

For variations of the Hamiltonian with respect to η, in the variation of kinetic energy it is important to realise that ϕ actually depends on η since ϕ is the potential at the surface. Hence, for given variation $\delta\eta$, to keep ϕ fixed at the varied surface, we also get a contribution from the induced change $\delta\phi = W\delta\eta$. To compensate this, in taking variations of η at fixed ϕ we have

$$\left\langle \delta_\eta\mathcal{K}, \delta\eta\right\rangle = \mathcal{K}\left(\eta+\delta\eta, \phi - W\delta_\eta\right) = \left\langle \bar{\delta}_\eta\mathcal{K}, \delta\eta\right\rangle - \left\langle \delta_\phi\mathcal{K}, W\delta\eta\right\rangle$$

where we denote by $\bar{\delta}_\eta$ the total variation with respect to η, allowing ϕ to change. Since

$$\bar{\delta}_\eta\mathcal{K} = \frac{1}{2}|\nabla\Phi|^2_{z=\eta} = \frac{1}{2}\left[(\phi_x - \eta_x W)^2 + W^2\right]$$