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# Evolution of breaking waves on sloping beaches

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ABSTRACT

This study investigates the evolution of breaking waves on sloping beaches. The motion of water particles is formulated in the Lagrangian framework that uses label and time as the independent variables. A classic perturbation scheme is employed for solutions. In this paper, a Lagrangian solution up to the first order in term of the beach slope is presented. In the solution the wave breaking criteria commonly used are adopted: waves break when the horizontal particle velocity at the wave crest is equal or greater than the wave celerity, or when a vertical tangent profile occurs at the free surface. The continuous wave deformations and breaking calculated from the Lagrangian solution are presented. A series of experiments are also conducted in a laboratory wave tank for observing wave evolution and verifying the Lagrangian solution. It shows that the Lagrangian solution and the experiment data, including the plunging, post-plunging, spilling and post-spilling breaking waves, agree reasonably well.



## 1. Introduction

As a wave propagates to shallower water depth on a beach, its wave height increases; its wavelength decreases; and its profile becomes steeper, asymmetric, unstable, and finally it breaks. In the past many investigators, such as Sverdrup and Murk (1944) and Méhauté and Webb (1964), use the conservation of energy flux to solve the wave transformation on a sloping bottom, which is approximated by a bottom of a finite number of steps. In this situation, the effect of the bottom slope could not be fully exhibited and the wave breaking profile is not correctly depicted by the analytical solution (Chen et al., 2004, 2005). Lewy (1946), Stoker (1947) and Lowell (1949) obtain the linearized solutions of the shallow water equations on beaches of uniform slope for certain beach slopes only, not for arbitrary bottom slopes. Peters (1952) proposes an integral solution for the surface wave propagating on the sloping bottom, and the solution is later calculated by Ehrenmark (1998). Keller (1958) derives an alternative linear form for progressive waves on an uneven bottom; however, the solution is expressed in an implicit form and is restricted to short waves in deep-water on a gentle sloping beach. Clearly, most of the above-mentioned theories are not able to present the complete wave transformation on a sloping bottom.

Up to now, few investigators have presented analytic solutions for the wave transformation on a sloping beach. Biesel (1952) suggests an approximation method to account for the normal incident waves propagating on a sloping bottom where the bottom slope $\alpha$ is first considered in the velocity potential as a perturbation parameter. The two-parameter perturbation expression based on the wave steepness $\varepsilon$ and the bottom slope $\alpha$ (correct to $\varepsilon^1\alpha^3$) is used to derive the expression of the velocity potential (Chen et al., 2005). These solutions are able to simulate the subsequent wave profiles before wave-breaking point. However, the behaviors of wave steepening and overturning during and after breaking remain absent and unclear. The objective of this study is to present an analytical solution in the Lagrangian framework for investigating the evolution of the breaking and post-breaking waves.

It is well known that in the Eulerian description the free surface $y = \eta$ can be expressed as a Taylor series expanded at $y = 0$, which implicitly assumes that the surface profile is supposed to be differentiable. This assumption limits the applicability of the Eulerian description at the near-break condition. On the other hand, in the Lagrangian description the free surface is parameterized by a fixed particle label in the spatial domain. It is advantageously appropriate in the Lagrangian description for limiting free surface motion (Chen and Hsu, 2009; Chen et al., 2005; Hsu et al., 2009a,b; Naciri and Mei, 1992). Thus, the Lagrangian solutions are expected to be able to describe the wave breaking processes much more precisely than the Eulerian solutions.

In this study, we adopt the Lagrangian solution by Chen et al.(2007, 2012) for investigating the wave profile prior to wave breaking point. We also further adopt the well-known wave breaking criterion that the horizontal particle velocity at the wave crest must be at least equal to the wave phase speed (i.e. kinematics stability parameter) for the initial breaking stage to occur. The equation of particle motion is

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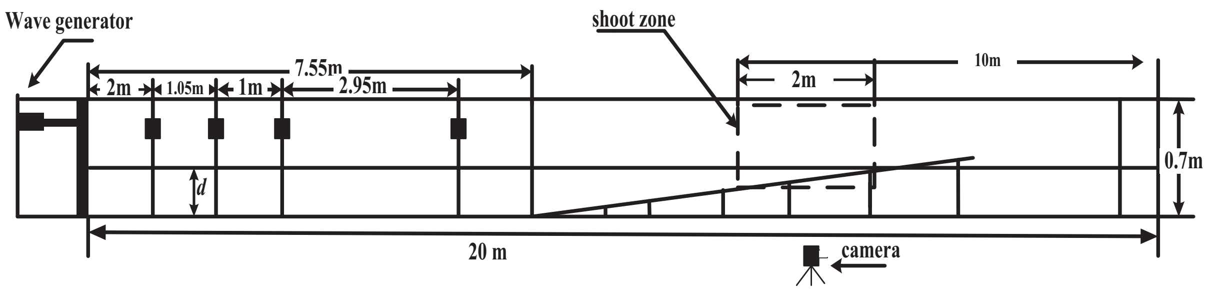


**Fig. 1.** Experimental setup.

**Table 1**
Experimental wave parameters.

| Initial water depth $d$(cm) | Bottom slope $\alpha$ | Wave period $T$(s) | Incident wave height $H_i$(cm) | $\lambda_0 a_0$ | Theory | Measured | Theory | Measured | Surf similarity parameter $\varsigma$ by Galvin(1968) | Case no. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Breaking wave height $H_b$(cm) | | Breaking water depth $d_b$(cm) | | | |
| 37.6 | 1/20 | 1.00 | 4.08 | 0.082 | 4.96 | 4.52 | 3.13 | 3.0 | 0.298 | (1) |
| | | 0.99 | 5.97 | 0.123 | 6.77 | 6.31 | 4.09 | 3.99 | 0.244 | (2) |
| | | 1.00 | 7.71 | 0.155 | 8.53 | 8.44 | 5.35 | 4.98 | 0.217 | (3) |
| | | 1.49 | 3.93 | 0.036 | 5.21 | 4.76 | 3.56 | 3.31 | 0.452 | (4) |
| | | 1.50 | 7.61 | 0.068 | 9.11 | 8.66 | 5.73 | 5.28 | 0.327 | (5) |
| | | 2.01 | 4.02 | 0.020 | 5.61 | 5.23 | 4.05 | 3.74 | 0.625 | (6) |
| | | 1.99 | 6.17 | 0.031 | 8.02 | 7.73 | 5.42 | 5.11 | 0.499 | (7) |
| | | 2.01 | 8.17 | 0.041 | 10.22 | 10.05 | 6.64 | 6.21 | 0.439 | (8) |
| | | 2.49 | 4.22 | 0.014 | 6.08 | 6.06 | 4.51 | 4.23 | 0.784 | (9) |
| | | 2.99 | 4.3 | 0.010 | 6.55 | 6.38 | 4.82 | 4.51 | 0.968 | (10) |
| | | 3.01 | 6.84 | 0.015 | 9.45 | 9.11 | 6.69 | 6.43 | 0.774 | (11) |
| | | 3.01 | 9.38 | 0.21 | 12.29 | 11.88 | 8.34 | 8.04 | 0.661 | (12) |
| 40 | 1/10 | 0.80 | 6.40 | 0.201 | 6.80 | 6.44 | 3.97 | 3.92 | 0.390 | (13) |
| | | 0.81 | 8.38 | 0.257 | 8.60 | 8.76 | 4.93 | 5.01 | 0.344 | (14) |
| | | 1.00 | 6.25 | 0.126 | 7.07 | 6.62 | 4.27 | 4.2 | 0.483 | (15) |
| | | 1.21 | 6.07 | 0.083 | 7.04 | 6.58 | 4.44 | 4.43 | 0.586 | (16) |
| | | 1.51 | 4.03 | 0.036 | 5.36 | 5.46 | 3.67 | 3.78 | 0.904 | (17) |
| | | 1.51 | 6.04 | 0.053 | 7.53 | 7.96 | 4.89 | 5.21 | 0.738 | (18) |
| | | 1.99 | 4.15 | 0.021 | 5.78 | 5.38 | 4.18 | 4.2 | 1.209 | (19) |
| | | 2.52 | 4.37 | 0.014 | 6.27 | 6.36 | 4.68 | 4.92 | 1.553 | (20) |
| | | 3.01 | 4.38 | 0.010 | 6.52 | 6.97 | 5.02 | 5.32 | 1.916 | (21) |
| | | 3.01 | 7.06 | 0.016 | 9.70 | 9.27 | 6.95 | 7.16 | 1.510 | (22) |

also derived using the free falling concept to capture the post-breaking free surface profiles. Later, we will present and discuss the breaking wave profiles in detail including the overhanging characteristics of the plunging and spilling breakers. Laboratory experiments in a wave tank are also conducted for validating the theoretical breaking and post-breaking surface profile on sloping bottoms.

## 2. Experimental setup and procedures

The purpose of this experiment is to investigate the wave breaking process for progressive waves on a sloping bottom. These experimental data are also used to validate the present theoretical solution. Wave breaking profiles can be obtained by the theoretical solution and experimental measurements (consecutive photographs). The experimental measurements are carried out in a glass-walled wave tank with the dimension of 20.0 m(l) × 0.5 m(w) × 0.7 m(h) at the Tainan Hydraulics Laboratory of the National Cheng Kung University, Taiwan. A high-speed camera is set up in front of the glass wall at 10–11 m from the wave generator to successively capture the surface wave images in the tank. This method also allows successively capturing the breaking wave profiles. Four wave gauges are set up at the locations of 2 m, 3.05 m, 4.05 m, and 7 m from the wave generator. The entire experimental setup is schematically shown in Fig. 1. A piston-type wave generator is used for generating monochromatic waves. Images of the wave profiles are captured by a high-speed camera. (MS55k2, Canadian Photonic Labs Inc.) The camera resolution is 1280 × 1080 pixels and the maximum framing rate is 1020 frames per second (fps). A transparent acrylic-plastic sheet (0.9 m × 0.625 m) is calibrated at 1-mm intervals in 5 mm × 5 mm grids. It serves as virtual grids in the picture. The experiments are conducted at two different bottom slopes ($\alpha = 1/10$ and 1/20), two different initial water depths ($d = 37.6$ cm and 40 cm) and several wave periods ($T = 0.80$–3.01 s). The incident wave height $H_i$ ranges from 3.93 cm to 9.38 cm. All of the experimental wave parameters are shown in Table 1, where $H_b$ denotes the breaking wave height, and $d_b$ the breaking water depth. From the experimental induction, the breaker

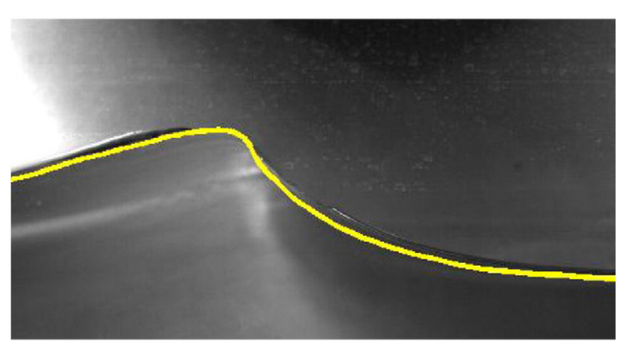

**Fig. 2.** Wave profile from snapshot.