Coastal Engineering 90 (2014) 23–39

Contents lists available at ScienceDirect

Coastal Engineering

journal homepage: www.elsevier.com/locate/coastaleng

A simple general expression for longshore transport of sand, gravel and shingle

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ARTICLE INFO

Article history:
Received 3 January 2014
Received in revised form 17 April 2014
Accepted 22 April 2014
Available online 16 May 2014

Keywords:
Longshore transport
Littoral drift
Sand, gravel and shingle beaches

ABSTRACT

Longshore transport of sand, gravel and shingle has been studied using field and laboratory data over a wide range of conditions. A detailed model (CROSMOR) for cross-shore and longshore sediment transport has been used to determine the effects of wave period, grain size, beach/surf zone slope and type of waves (wind waves or swell waves). The longshore transport was found to be proportional to wave height to the power 3.1 ($\approx H^{3.1}$), to grain size to the power −0.6 ($\approx d_{50}^{-0.6}$) and to beach slope to the power 0.4 ($\approx \tan\beta^{0.4}$). Regular swell waves yield much larger (factor 1.5) longshore transport rates than irregular wind waves of the same height. It is proposed to take this effect into account by a swell correction factor. Based on all results, a new simple and general (dimensionally correct) expression for longshore transport of sand, gravel and shingle beaches with grain sizes between 0.1 and 100 mm has been derived. Short-term and long-term field data of sand, gravel and shingle have been used for verification. In most cases the predicted longshore transport rates are within a factor of 2 of the measured values. The CERC and Kamphuis formulas have also been tested.

1. Introduction

The prediction of reliable estimates of longshore sediment transport is of considerable practical importance in coastal engineering. An example is the evaluation of sediment budgets for coastal areas with and without structures (breakwaters, groynes). Another example is the long-term stability of beach protections and beach nourishments. Most research on longshore transport has concentrated on sand sized sediment, but research on longshore transport along gravel/shingle beaches, which are quite common along mid- and high-latitude (formerly glaciated) parts of the world, has been very limited.

The most widely used formula for longshore transport (LT) is the CERC equation (Shore Protection Manual, US Army Corps of Engineers, 1984). This method is based on the principle that the longshore transport rate (LT, incl. bed load and suspended load) is proportional to longshore wave power P per unit length of beach; LT = K P, with K = calibration coefficient. The CERC formula has been calibrated using field data from sand beaches. The CERC formula does not account for particle size and beach slope. It is only valid for sandy conditions.

The effects of particle diameter and bed slope have been studied systematically by Kamphuis (1991), resulting in a more refined equation for longshore sediment transport. The Kamphuis formula is valid for sand beaches, but is most likely not valid for gravel and shingle beaches. The Kamphuis formula was found to give the best agreement between computed and measured transport rates based on the work of Schoonees and Theron (1993, 1996). Recently Mil-Homens et al. (2013) have made a re-evaluation of the Kamphuis formula based on an extensive set of 250 data points. Most of the data points are in the sand range (<0.6 mm) and low transport range (mild wave conditions). Datasets of gravel and shingle beaches, which is a focus point of the present study, have not been used. The modified Kamphuis 2013 formula will also be used in the present study.

Van Wellen et al. (2000) have evaluated various formulae for coarse-grained beaches, but most of these formulae are not suitable for sand beaches. The formula of Damgaard and Soulsby (1997, 2005) for longshore bed load transport of coarse materials performed rather well. However, this formula does not give the longshore suspended load transport for sand beaches. Tomasicchio et al. (2013) proposed a general set of equations for longshore transport of coarse to fine sediments. They started from available expressions for very coarse materials as used for breakwaters. These expressions were extended to the sand range by a fitting procedure using 245 data points split in various subintervals. An independent verification was not done.

Herein, a detailed process-based model (CROSMOR) has been used to compute the longshore transport rates along sand, gravel and shingle beaches. This model was tested using a small but good-quality field dataset (22 data points) of sand and shingle beaches. Then, the model was applied to study the systematic effects of particle size, wave period and profile shape on the longshore transport process. The CROSMOR results have been parameterized and implemented in a new simple formula, which is a modification of the longshore transport formula presented by Van Rijn (2002). The new formula is now dimensionally

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http://dx.doi.org/10.1016/j.coastaleng.2014.04.008
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correct and valid for the size range from sand to shingle and cobbles (0.1 to 100 mm). The effects of additional currents due to tide and wind can be easily included. Short term and long term field and laboratory data have been used for (independent) verification of the new expression.

2. Field data analysis

2.1. Sand beaches

Schoonees and Theron (1993, 1996) have made an extensive inventory of the available datasets (about 270) of longshore sand transport rates from a variety of sites around the world. Most datasets refer to mild wave conditions with offshore wave heights smaller than about 2 m and bed material in the sand range of 0.2 to 0.6 mm. The lack of data in the storm range was partly solved by the special storm measurements performed by the US Army Corps of Engineers (USACE) from the Field Research Facility at the Duck site (USA) in the years of 1995 to 1998 (Miller, 1999). These latter datasets with longshore transport rates in conditions with offshore wave heights up to 4 m have also been used in this study.

In the present study 16 reliable datasets (only sand) from 6 sites in the USA have been applied to analyse the longshore sand transport process. All data in the sand range (0.15 to 0.45 mm) were taken from the original data reports and papers and checked for reliability. The datasets from Duck (USA) and Indian Rocks (USA) were taken because they represent the two extreme ends (high breaking waves up to 3.2 m and low waves of 0.3 m) of the transport range. The other datasets are in the intermediate transport range (waves in the range of 0.5 to 2 m), see Table 1. All datasets satisfy the criteria of available wave conditions (height, period and angle), transport rates, beach slopes and grain sizes. Most datasets used herein refer to short-term measurements using direct sampling methods or short-term volume changes at USA field sites. As regards long-term volume changes, only one case with a predominant wave direction has been considered (Leadbetter Beach, USA). Most often, the longshore transport rates are given as bulk volumetric rates ($Q_{t,volume}$ in m^3 per day). This value has been converted to the mass transport rate ($Q_{t,mass}$ in kg/s) by using $Q_{t,mass} = (1 - p)\,\rho_s\, Q_{t,volume}$ with $p =$ porosity factor (0.4 for sand and 0.45 for shingle) and $\rho_s =$ sediment density ($= 2650$ kg/m^3). The measured transport rates are in a very wide range of 0.3 to 900 kg/s (factor 3000!).

The characteristics of the 6 field sites are:

- South Lake Worth, Florida, USA (1953); low wave and microtidal conditions (swell); medium coarse sand bed of 0.4 to 0.6 mm; data based on bypassing rate of sand pumping plant (Watts, 1953);
- Leadbetter Beach, California, USA (1981); mild wave conditions (swell); fine sand of 0.2 to 0.25 mm; data based on morphological volume changes along beach over about 1 year (Gable, 1981);
- Lake Michigan, Wisconsin, USA (1975); low wave energy conditions; medium fine sand of 0.2 to 0.3 mm; data based on trap samplers (Lee, 1975);
- Price Inlet, South Carolina, USA (1977); low wave energy conditions (swell); medium fine sand of 0.2 to 0.25 mm; data based on trap samplers (Kana and Ward, 1980; Kana et al., 1977);
- Indian Rocks Beach, Florida, USA (1999); very low wave-energy and microtidal conditions; medium coarse sand bed of 0.35 mm; data based on short-term morphological volume changes (<1 day) (Wang and Kraus, 1999);
- Duck site, USA (1985 and 1995–1998); medium to high waves and microtidal conditions; fine sand bed of 0.15 to 0.2 mm; data based on sampling using streamer traps during swell conditions in September 1985 (Kraus and Dean, 1987); electronic concentration sampling during storm conditions (Miller, 1999).

This relatively small but reasonably good-quality dataset was used to establish the relationship between wave height, wave incidence angle and longshore transport. It is realized that the applied sand dataset is small, but in Section 3.2 it will be shown that this dataset actually is quite representative. More data of the same will give more scatter, but this may help to better define the envelope of variation in the data.

The measured total longshore sand transport rates (16 cases from 6 field sites) are plotted in Fig. 1 as function of the parameter $W = (H_{s,br})^3 \sin(2\theta_{br})$. The power of the wave height is found to be about 3 based on two extreme cases with very low waves (Indian Rocks site) and high waves (Duck site). Most transport rates are within a factor of 2 of the plotted trend line. The trend line can be represented by (see also Van Rijn, 2002):

$$Q_{t,mass} = K_{sand}\left(H_{s,br}\right)^{3} \sin(2\theta_{br}) \tag{1}$$

with: $Q_{t,mass} =$ longshore sand transport (in kg/s; dry mass); $H_{s,br} =$ significant wave height at breaker line (in m); $\theta_{br} =$ wave incidence angle (to shore normal) at breaker line (degrees) and $K_{sand} = 40$ (kg/s/m^3).

Eq. (1) is valid for sand in the range of 0.15 to 0.42 mm and beach slopes in the range of 0.02 to 0.1. The complete dataset for sand is too small to detect any influence of particle size and/or beach slope.

Table 1
Longshore transport data of field sites (sand) in USA.

Field sites USA	d_{50} (mm)	tanβ (−)	$H_{s,br}$ (m)	θ_{br} (°)	T_p (s)	$Q_{t,mass}$ (kg/s)	Type of load
Lake Worth, WTTS1 1952	0.42	0.03	0.55	17	7	5	Total load
Lake Michigan, 1978	0.25	0.08	0.65	25	4	4.3	Only suspended
Leadbetter, LBB17 1981	0.22	0.046	0.855	6	11	13.5	Total load
Leadbetter, LBB32 1981	0.22	0.019	1.77	8	11.9	197	Total load
Price Inlet, BU2 1977	0.22	0.018	0.7	9	9.5	7.4	Only suspended
Price Inlet, CA1 1977	0.22	0.027	0.8	9	9.2	16.4	Only suspended
Duck, 14 November 95	0.15–0.2	0.025	1.70	10	8	144	Only suspended
Duck, 11 March 96	0.15–0.2	0.025	2.40	10	7	483	Only suspended
Duck, 27 March 96	0.15–0.2	0.025	1.85	19	7	152	Only suspended
Duck, 2 April 96	0.15–0.2	0.025	1.75	19	7	180	Only suspended
Duck, 1 April 97	0.15–0.2	0.025	2.85	16	9	395	Only suspended
Duck, 19 October 97	0.15–0.2	0.025	3.20	18	10	730	Only suspended
Duck, 4 February 98	0.15–0.2	0.025	3.10	19	11	920	Only suspended
Duck, 5–7 September 85	0.2	0.025	1.05	3	9.5	4	Only suspended
Indian Rocks, Run 4 1999	0.35	0.09	0.29	13.4	3.6	0.33	Total load
Indian Rocks, Run 5 1999	0.35	0.13	0.40	19.7	3.0	0.95	Total load

$d_{50} =$ particle size; tanβ = beach/surf zone slope; $H_{s,br} =$ significant wave height at breaker line; $\theta_{br} =$ wave angle to shore normal at breaker line; $T_p =$ peak wave period.