Coastal Engineering 94 (2014) 1–9

Contents lists available at ScienceDirect

Coastal Engineering

journal homepage: www.elsevier.com/locate/coastaleng

The application of a Godunov-type shock capturing scheme for the simulation of waves from deep water up to the swash zone

Ali Shirkavand *, Peyman Badiei

School of Civil Engineering, College of Engineering, University of Tehran, P.O. Box 11365-4563, Tehran, Iran

ARTICLE INFO

Article history:
Received 11 March 2014
Received in revised form 20 June 2014
Accepted 18 August 2014
Available online 15 September 2014

Keywords:
Non-hydrostatic wave model
Godunov-type shock-capturing scheme
Surf zone model
Wave breaking
Swash motion

ABSTRACT

A two-dimensional vertical (2DV) non-hydrostatic boundary fitted model based on a Godunov-type shock-capturing scheme is introduced and applied to the simulation of waves from deep water up to the swash zone. The effects of shoaling, breaking, surf zone dissipation and swash motions are considered. The application of a Godunov-type shock-capturing algorithm together with an implicit solver on a standard staggered grid is proposed as a new approach in the 2DV simulation of large gradient problems such as wave breaking and hydraulic jumps. The complete form of conservative Reynolds averaged Navier–Stokes (RANS) equations are solved using an implicit finite volume method with a pressure correction technique. The horizontal advection of the horizontal velocity is solved by an explicit predictor–corrector method. Fluxes are predicted by an exact Riemann solver and corrected by a downwind scheme. A simple total variation diminishing (TVD) method with a monotonic up-stream-centered scheme for conservation laws (MUSCL) limiter function is employed to eliminate undesirable oscillations across discontinuities. Validation of the model is carried out by comparing the results of the simulations with several experimental test cases of wave breaking and run-up and the analytical solution to linear short waves in deep water. Promising performance of the model has been observed.

1. Introduction

In recent years, coastal engineers and researchers have been widely engaged in the numerical simulation of near-shore processes. Many of such attempts are based on Boussinesq equations that are dependent on empirical parameters for the simulation of wave breaking and dissipation in the surf zone. Madsen et al. (1997) and Schaffer et al. (1993) employed the concept of surface rollers and Karambas and Koutitas (1992), Zelt (1991) and Kennedy et al. (2000) adopted an artificial viscosity for this purpose.

Several methods with different degrees of complexity have been proposed for the modeling of free surface flow. Lin and Liu (1998) and Bakhtyar et al. (2009) used Volume of Fluid (VOF), Christensen and Rolf (2001) applied Marker and Cell (MAC) and Wang et al. (2009) adopted the Level Set Method for the simulation of wave propagation, breaking, dissipation and run-up. These types of models can handle complex water surface profiles in cases such as plunging breakers, but they are limited by high computational expenses (Lin and Li, 2002).

Development of non-hydrostatic free surface models which could be applied to a relatively large coastal domain has been the target of many research endeavors. Free surface level is obtained by vertical integration of continuity equation and applying kinematic boundary condition. The advantage of such models is the need for fewer vertical layers in the simulation of free surface elevation, run-up and run-down with a relatively low computational cost in comparison with VOF, MAC and level set methods. Zijlema and Stelling (2005), Yuan and Wu (2006), Ahmadi et al. (2007) and Badiei et al. (2008) have used this approach to simulate short wave propagation up to the incipient breaking.

Non-hydrostatic models have been further developed to simulate near shore wave processes. Zijlema and Stelling (2008) pioneered the efforts along this line and proposed a model capable of considering wave breaking, dissipation and run-up, using only two vertical layers. Yamazaki et al. (2009) derived a set of modified shallow water equation by introducing the non-hydrostatic pressure into the depth averaged momentum equation. Applying this method, they were able to simulate weak wave breaking and run-up. Ai and Jin (2012) introduced a multi-layer non-hydrostatic model using the conservative form of the momentum equations. They simulated wave breaking and run-up successfully. All the models mentioned above use staggered grids in horizontal direction. Ma et al. (2012) proposed a non-hydrostatic shock-capturing model with a collocated grid in horizontal and Keller-Box grid in vertical direction. They used a Godunov-type scheme with HLL Riemann approximation for horizontal fluxes. They were able to simulate non-linear wave propagation, weak breaking and run-up.

All of the models described above are able to simulate wave breaking with a few vertical layers (<10). However the test cases used for comparison are limited to weak wave non-linearity. Smit et al. (2013)

* Corresponding author.
E-mail address: shirkavand@ut.ac.ir (A. Shirkavand).

http://dx.doi.org/10.1016/j.coastaleng.2014.08.010
0378-3839/

adopted the hydrostatic front approximation (HFA) method proposed by Tonelli and Petti (2010), for incipient breaking criterion and subsequent dissipation and managed to simulate wave breaking and dissipation with a limited number of layers (less that 5).

Most of the shock-capturing methods of Godunov-type are designed for collocated grids. Two alternatives are reported in the literature, which are able to capture sharp shocks like hydraulic jumps and wave breaking on a staggered grid. Zijlema et al. (2011) used the conservative form of momentum equation and estimated the water depth at velocity point by an upwind estimator, as suggested by Stelling and Duinmeijer (2003), and applied MacCormack predictor–corrector technique for horizontal advection of horizontal velocities. Yekta and Banihashemi (2011) is the second case in which the same upwind estimator and Godunov-type shock-capturing for horizontal advection were used to solve non-linear shallow water equation.

In this paper a 2DV non-hydrostatic shock-capturing model is developed to solve the complete form of RANS, using an implicit finite volume method. Here we have extended the Godunov-type shock-capturing method proposed by Yekta and Banihashemi (2011) to a 2DV model. A boundary fitted standard staggered grid is used, which makes the model suitable for irregular bed and surface level changes.

In order to simulate wave propagation and breaking by a limited number of vertical layers, a non-hydrostatic pressure correction as suggested by Badiei et al. (2008) is employed at the top layer and a hydrostatic front approximation (HFA) is applied.

In Section 2, the governing equations together with boundary conditions are introduced. Numerical techniques are presented in Section 3, and in Section 4 the results of the simulations are validated by comparing against experimental data on wave breaking and run-up and analytical solution to linear deep water waves.

2. Governing equations

Two dimensional vertical incompressible Reynolds averaged Navier–Stokes equations are employed. Substituting the pressure $P = -\rho g(z-\eta) + \rho q$, the following sets of equations are obtained:

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uw}{\partial z} + g\frac{\partial \eta}{\partial x} + \frac{\partial q}{\partial x} = \frac{\partial}{\partial x}\left(\nu_h \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial z}\left(\nu_v \frac{\partial u}{\partial z}\right) \tag{2}$$

$$\frac{\partial w}{\partial t} + \frac{\partial wu}{\partial x} + \frac{\partial w^2}{\partial z} + \frac{\partial q}{\partial z} = \frac{\partial}{\partial x}\left(\nu_h \frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial z}\left(\nu_v \frac{\partial w}{\partial z}\right). \tag{3}$$

Here q represents the non-hydrostatic part of the pressure, η is the surface level, ρ is the fluid density, g is the gravitational acceleration, t is the time and u and w are respectively the horizontal and vertical velocity components. ν_h and ν_v are respectively the horizontal and vertical eddy viscosities. Horizontal eddy viscosity is obtained by applying the Smagorinsky (1963) method and vertical eddy viscosity is solved by $k-\varepsilon$ method. For a small number of vertical layers (less that 5) a constant vertical eddy viscosity is applied ($\upsilon_v = 10^{-4}\text{m}^2/\text{s}$).

Kinematic boundary conditions at the free surface read as:

$$\frac{\partial \eta}{\partial t} + u\frac{\partial \eta}{\partial x} = w|_{z=\eta} \tag{4}$$

and at the bed:

$$-u\frac{\partial h}{\partial x} = w|_{z=-h} \tag{5}$$

where h is the still water depth.

The dynamic free surface boundary condition, considering atmospheric pressure at the surface level is:

$$P_a = -\rho g(z-\eta) + \rho q \;\Rightarrow\; q = \frac{P_a}{\rho} \cong 0. \tag{6}$$

Free surface level η is an unknown that is calculated by integrating continuity equation over depth and applying kinematic free surface and bottom boundary conditions. The resulting equation reads as:

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}\int_{-h}^{\eta} u dz = 0. \tag{7}$$

Incoming flow is best introduced at the lateral boundaries by horizontal velocity or volume flux at each layer. Moving shoreline is implemented at the shore boundary.

3. Numerical methods

The computational domain is discretized by a boundary fitted standard staggered grid with an equal number of layers, as shown in Fig. 1. The layer thickness is obtained by $d_k = f_k.\, D = f_k.\, (\eta + h)$ with $0 < f_k < 1$ and $\sum_k f_k = 1$.

As shown in Fig. 2, pressure and other scalar parameters are defined at cell centers and velocities are defined at the edges of each cell. Surface and bottom levels are respectively defined at the center of the outer edges of top and bottom cells of a water column.

In order to determine the layer thickness at the vertical edges of each cell, we need the total water depth at that edge. This is calculated by an upwind estimator according to Zijlema et al. (2011):

$$\hat{D}_{i+\frac{1}{2}} = \begin{cases} \eta_i + \min(h_i, h_{i+1}) & \text{if } U_{i+\frac{1}{2}} > 0 \\ \eta_{i+1} + \min(h_i, h_{i+1}) & \text{if } U_{i+\frac{1}{2}} < 0 \\ \max(\eta_i, \eta_{i+1}) + \min(h_i, h_{i+1}) & \text{if } U_{i+\frac{1}{2}} = 0 \end{cases} \quad , \quad \hat{d}_{i+\frac{1}{2},k} = f_k \cdot \hat{D}_{i+\frac{1}{2}}. \tag{8}$$

Here U is the depth averaged velocity calculated at the edge of each column, D is the total depth at the same edge and d is the layer thickness.

3.1. Numerical discretization and solution algorithm

The projection method with a pressure correction technique proposed by van Kan (1986) and time splitting algorithm are used for the solution of governing equations. The set of equations are solved in two steps. In the first step, momentum equation is solved considering advection, diffusion, surface level gradient, bottom friction and known dynamic pressure gradient (from the previous time step). As a result of this step, intermediate velocities are obtained. In the next step, in a

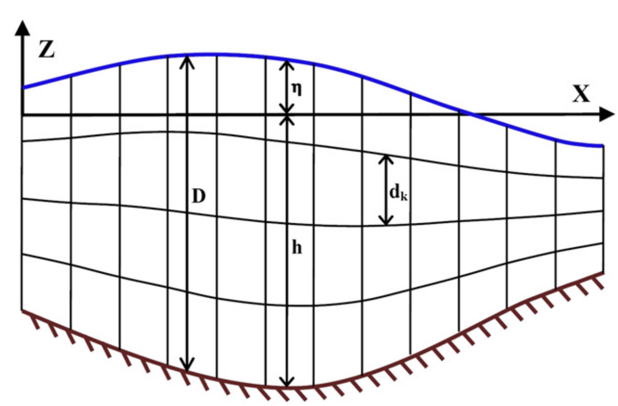

Fig. 1. Boundary fitted grid system deployed for 2DV calculations.