Coastal Engineering 94 (2014) 63–79

Contents lists available at ScienceDirect

Coastal Engineering

journal homepage: www.elsevier.com/locate/coastaleng

Importance of second-order wave generation for focused wave group run-up and overtopping

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ARTICLE INFO

Article history:
Received 12 March 2014
Received in revised form 10 August 2014
Accepted 11 August 2014
Available online 26 September 2014

Keywords:
Wave-maker theory
Spurious error wave
Run-up
Overtopping
Focused wave groups
Boussinesq numerical wave tank

ABSTRACT

Background: Focused wave groups offer a means for coastal engineers to determine extreme run-up and overtopping events.
Research purpose: This work examines numerically the importance of second-order accurate laboratory wave generation for NewWave-type focused wave groups generated by a piston-type paddle generator, and interacting with a plane beach and a seawall in a wave basin.
Methods: The numerical wave tank is based on the Boussinesq equations for non-breaking waves, and the non-linear shallow water equations for broken waves. During the model validation, good agreement is achieved between the numerical predictions and laboratory measurements of free surface elevation, run-up distances and overtopping volumes for the test cases driven by linear paddle signals. Errors in run-up distance and overtopping volume, arising from linear wave generation, are then assessed numerically by repeating the test cases using second-order accurate paddle signals.
Results: Focused wave groups generated using first-order wave-maker theory are found to be substantially contaminated by a preceding long error wave, resulting in erroneously enhanced run-up distances and overtopping volumes.
Conclusions: Thus, the use of second-order wave-maker theory for wave group run-up and overtopping experiments is instead recommended.

1. Introduction

Accurate prediction of coastal wave run-up and overtopping is very important in scenario-driven analysis of likely flood events (see e.g. (Kobayashi, 1999), (Borthwick, 2009), (Baldock et al., 2012)). Considerable effort has been put into deriving empirical run-up and overtopping formulae (see e.g. (Hunt, 1959), (Hedges and Mase, 2004), (Hedges and Reis, 2004), (Allsop et al., 2005), (De Rouck et al., 2005), (Burcharth and Hughes, 2006), (Pullen et al., 2007)), supplemented by a great number of wave tests in different flumes and coastal basins (see e.g. (Pearson et al., 2002), (J. W. et al., 2009) and (Hunt-Raby et al., 2011)). There have also been major developments in coastal wave simulation methods, some based on the non-linear shallow water equations (e.g. (Hu et al., 2000), (Hubbard and Dodd, 2002)), Boussinesq-type equations (see for example (Fuhrman and Madsen, 2008) or review documents by (Dingemans, 1997), (Kirby, 1997), (Madsen and Schäffer, 1999), (Kirby, 2003), (Brocchini, 2013)), hybrid Boussinesq-shallow flow equations ((Watson et al., 1994), (Borthwick et al., 2006a), (Tonelli and Petti, 2009), (Tissier et al., 2011), (Orszaghova et al., 2012), (McCabe et al., 2013)), potential flow theory (Fructus and Grue, 2007), and the Navier–Stokes equations (see e.g. (Hsiao and Lin, 2010) for volume-of-fluid method, (Ingram et al., 2009) for free surface tracking implementation, and (Rogers and Dalrymple, 2008) for a smoothed particle hydrodynamics solver).

For almost 20 years, focused wave groups have been increasingly used by offshore engineers to represent the average shape of the extreme event in a Gaussian sea state ((Tromans et al., 1991), (Jonathan and Taylor, 1997), (Taylor and Williams, 2004)). Pioneering laboratory experiments on focused wave groups have been carried out in water of uniform depth ((Rapp and Melville, 1990), (Baldock et al., 1996) and (Johannessen and Swan, 2001)), demonstrating that ocean waves are dispersive and can evolve into transient, localised but energetic groups that focus in shallow coastal waters (Baldock, 2006). It is also plausible that a similar focused-wave analysis could be useful in assessing storm-induced wave run-up maxima at beaches and overtopping volumes at coastal defences. Focused wave group laboratory tests have the advantage that they are quick to perform, with all important data obtained before any waves reflected at the coast reach the

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http://dx.doi.org/10.1016/j.coastaleng.2014.08.007
0378-3839/

paddle, thus avoiding the gradual contamination of flumes by long wave reflections. However, it should be noted that there remain several important questions to be answered regarding the applicability of focused wave groups as design waves for extreme storm events at the coast. In reality, wave run-up at beaches and sea defence overtopping are strongly influenced by the preceding swash motions, and it is not necessarily the case that the peak run-up or overtopping is associated with the largest wave. Moreover, the near-shore wave energy spectrum is not the same as the corresponding offshore spectrum. Even so, there is a sound rationale for investigating systematically the behaviour of focused wave groups in coastal waters.

The present paper examines a preliminary question concerning the order of accuracy required for the paddle signal used to generate focused waves in a basin or flume. In particular, we examine the importance of the correct reproduction of second-order bound components in focused wave groups, and study their influence on wave group run-up at a plane beach and overtopping of a seawall. To this end, results are compared from laboratory and numerical tests, the latter utilising linear and second-order wave generation methods. The numerical wave flume is based on the (Madsen and Sørensen, 1992) set of Boussinesq equations and the non-linear shallow water equations. As direct comparisons with laboratory experiments are carried out, waves are introduced into the numerical domain via an in-built moving piston wave-maker, which mimics a mechanical laboratory wave generator. Details of the numerical scheme are given in Orszaghova et al. (2012).

The paper is structured as follows. Section 2 provides an introduction to wave-maker theory, NewWave focused wave groups, and the numerical wave tank. Section 3 describes a numerical model investigation into focused wave group evolution over a flat bed, using linear and second-order wave generation. Major discrepancies are identified in the resulting wave forms, these arise from error waves when only first-order wave generation is used. The numerical model is validated against experimental measurements involving wave group run-up at a plane beach (Section 4) and overtopping of a trapezoidal wall (Section 5). In both sections, the effect of second-order wave generation on focused wave run-up distances and overtopping volumes is examined numerically. Section 6 summarises the major conclusions.

2. Methods and background theory

2.1. Laboratory wave generation

Piston paddles are often used for mechanical wave generation in shallow water laboratory flumes and basins. In practice, a suitable wave-maker theory is used to compute the paddle displacement time series used to control the horizontal motion of the paddle. The wave-maker theory for irregular waves utilises a Stokes-like perturbation technique, whereby the dependent variables (velocity potential ϕ, free surface elevation ζ, paddle displacement x_p) are expressed as a power series, and the boundary conditions at the free surface and at the wave-maker are expanded using Taylor series. This results in the original non-linear boundary value problem being expressed as an infinite set of ordered linear partial differential equations. First-order wave-maker theory considers the linearised problem, whose solution consists of the desired progressive harmonic waves and evanescent modes. Evanescent modes are local non-propagating disturbances that arise due to a uniform velocity field with depth being forced at the piston paddle, but die out away from the wave-maker. The relationship between the amplitude of the generated progressive wave and the amplitude of the paddle displacement is also derived, and is known as the Biésel paddle transfer function. Detailed descriptions of linear wave-maker theory are given by Dean and Dalrymple (1991) and Hughes (1993).

Full wave generation theory correct to second order, for normally propagating waves and applicable to both piston and hinged wave-makers, was derived by Schäffer (1996), who extended earlier analysis by Barthel et al. (1983). Schäffer's theory aims to suppress generation of second-order spurious free waves, also known as parasitic or error waves, which are unintentionally generated when linear paddle signals are used. A brief outline of Schäffer's theory follows. Using the superposition principle, the second-order problem is split into three sub-problems, each governed by the Laplace equation and a specific set of boundary conditions. The first sub-problem considers the wave in the absence of the wave-maker, and is solved to give the bound second-order sub- and super-harmonics. The other two sub-problems bear resemblance to the first-order problem, and give rise to second-order free waves. The second sub-problem solves to give second-order error waves owing to the linear paddle signal deviating from the mean paddle position, and second-order bound waves not satisfying the paddle boundary condition. The third sub-problem describes the compensating second-order free waves generated by the second-order paddle signal, which is chosen to cancel out the error waves from the second sub-problem. In this way only the appropriate bound second-order waves are generated. Note, however, that the evanescent modes from the second and third sub-problems do not cancel each other out.

Simplistically, the use of linear wave generation results in the following situation. The desired weakly non-linear waves have both linear energy-bearing components and higher, mostly second-order, bound components. If these higher harmonics are not accounted for by the paddle motion, the resulting wave field consists of the correct bound non-linear components and locally cancelling sum and difference components. These are free error waves but locally (at the paddle) cancel the necessary bound components. Since these free waves have different propagation speeds than the main linear waves in the desired wave group, they escape and contaminate the overall wave field.

2.2. Numerical wave tank

A one-dimensional numerical model of a shallow-water flume with an in-built piston paddle moving boundary wave-maker is used for all simulations in this work. The model is based on a set of enhanced Boussinesq equations derived by Madsen and Sørensen (1992) and the non-linear shallow water equations. Wave breaking is described approximately, by locally switching to the non-linear shallow water equations when specified threshold wave steepness is reached. Broken waves are described as bores. The moving shoreline is calculated as part of the solution, utilising a wetting and drying approach devised by Brufau et al. (2002). Detailed description of the model's characteristics, including numerical implementation, is given by Orszaghova et al. (2012). The model is suitable for simulating propagation of weakly dispersive waves and can additionally model any associated inundation, overtopping or inland flooding within the same simulation. Note that the in-built piston paddle wave-maker mimics a real-world laboratory wave-maker in that it moves according to a supplied paddle displacement time series calculated using appropriate wave-maker theory (see Section 2.1 above). The paddle operates on a local movable grid, which is Lagrangian on the paddle face and Eulerian away from the paddle. The governing equations are, however, evolved on a fixed mapped grid, and the newly calculated solution is transformed back onto the moving grid via a domain mapping technique. Inclusion of the paddle in the numerical code allows for simulations of complete shallow water laboratory experiments, including the wave generation process, by utilising the actual paddle displacement time series used in the laboratory.

Orszaghova et al. (2012) provide a detailed account of tests used to verify the numerical model, encompassing movement of the wet/dry front, wave generation by means of the numerical paddle, and discretisation of the governing equations. Orszaghova et al. (2012) also report preliminary validation of the code against a range of