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Generating–absorbing sponge layers for phase-resolving wave models

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The accurate generation and absorption of water waves in phase-resolving models are critical issues in representing nearshore processes. Here, we present a source function method for combined wave generation and absorption using modified sponge layers. This technique can be easily adapted to a wide variety of systems, and does not require the solution of Green's functions but rather the simpler knowledge of solutions for free waves. These solutions may be linear or nonlinear, regular or irregular, and generated waves can be made arbitrarily accurate through simple selection of sponge layer coefficients. Generating–absorbing sponge layer systems are shown to have a close correspondence to relaxation zones for wave generation if relaxation coefficients are chosen appropriately.

1. Introduction

The generation and absorption of waves at the boundary are important for the numerical simulation of Boussinesq and other water wave models. Relatively straightforward methods used by Nwogu (1993), Kennedy and Fenton (1997) and many others specified the incident wave information at the wavemaker boundary, with either no special treatment for reflected waves or approximate boundary conditions. These types of boundaries are compact and save on computational expense, but treatment for outgoing waves is by necessity approximate. Other techniques include various widely-used internal generation methods using either distributed or point sources in the governing equations (Chawla and Kirby, 2000; Larsen and Dancy, 1983; Skotner and Apelt, 1999; Wei and Kirby, 1999). Relaxation zones (Madsen et al., 2003), where an imposed solution is gradually transitioned to the governing equations over some distance, have been used both to generate and absorb waves in high accuracy Boussinesq models.

Associated with the wave generation problem is that of absorbing reflected or other waves that approach open boundaries. Here, by far the most common techniques are the various sponge layers (e.g., Chawla and Kirby, 2000; Larsen and Dancy, 1983) that extract mass, momentum, or both from the system, damping solutions to a steady-state with no waves. For nonlinear wave generation, both direct imposition of boundary fluxes and relaxation zones have been demonstrated to work, but accurate nonlinear wave generation using internal sources is quite difficult, and no good solutions exist.

This paper introduces and tests a combined generating–absorbing condition for phase-resolving wave models that is straightforward to apply to a wide variety of systems. The condition does not require derivation of Green's functions as with many internal generators (Chawla and Kirby, 2000) but instead requires a knowledge of free wave solutions, which are simpler to derive and are known for a wide range of equations. These free wave solutions may be linear or nonlinear, regular or irregular, and may be reproduced to arbitrary levels of accuracy. At the same time as the system generates waves, it absorbs outgoing signals in the same way as a typical sponge layer. Analytical and numerical tests show excellent performance for a range of conditions including irregular, nonlinear, wave generation.

2. Generating–absorbing sponge layers

The concept of sponge layers was introduced by Israeli and Orszag (1981), and is widely used to remove unwanted signals at the edge of domains, and prevents them from re-reflecting off open boundaries. For the present paper, it may be extended and written as

$$[\mathbf{A}_1]\left[\mathbf{a}_{,t}\right]+[\mathbf{L}_1]\left[\mathbf{a}_{,t}\right]+\text{other terms}=\omega_1[\mathbf{A}_1]\left[\mathbf{a}_{imp}-\mathbf{a}\right]+\omega_2[\mathbf{L}_1]\left[\mathbf{a}_{imp}-\mathbf{a}\right] \tag{2.1}$$

where $[\mathbf{a}](x,y,t)$ is the vector of variables (which would be $(\eta,U,V)^T$ for many Boussinesq-type systems where η is the surface elevation and (U,V) are the velocity variables, $\mathbf{a}_{,(-)}\equiv\partial\mathbf{a}/\partial(-)$, $\omega_1(x,y)$ and $\omega_2(x,y)$ are non-negative real damping coefficients. The matrix $[\mathbf{A}_1]$ contains algebraic multipliers of $[\mathbf{a}_{,t}]$ (e.g., 1 or h) while the matrix $[\mathbf{L}_1]$ contains spatial differential operators of $[\mathbf{a}_{,t}]$ (e.g., $h^2\partial^2/\partial x^2$). In other words, ω_1 may be thought of as modifying pure time derivative terms in the system, while ω_2 modifies mixed space–time terms. Together, they contain all time derivative terms that may be operated on by damping. Separate

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damping coefficients are used as it will be shown that there are advantages and disadvantages to using both forcings, depending on the situation.

The heart of the system is the vector of imposed quantities, $[\mathbf{a}_{imp}](x, y,z,t)$. For typical water wave implementations (Wei and Kirby, 1999) velocities would be damped towards zero, with perhaps elevation damping to a desired tide level, leading to a system which damps towards quiescence. However, damping to zero is not always necessary or even helpful. Here we specify that imposed quantities $[\mathbf{a}_{imp}](x,y,t)$ must be homogeneous solutions to the undamped system, i.e., Eq. (2.1) with $\omega_1 = \omega_2 = 0$. These imposed quantities are the desired waves to be generated, and may be linear or nonlinear, steady or unsteady.

By inspection, we see that if $[\mathbf{a}] = [\mathbf{a}_{imp}]$, the right hand side of Eq. (2.1) is zero and the desired free wave propagates identically to the undamped equations. However, if $[\mathbf{a}] \neq [\mathbf{a}_{imp}]$, the right hand side terms in Eq. (2.1) will gradually force the solution towards $[\mathbf{a}_{imp}]$ in the same way that a standard sponge layer forces velocities and/or elevations to zero. In this way, the system can generate the imposed waves at the same time as it damps other disturbances like reflected waves. Generation and absorbing zones are placed along the boundaries, and are the only locations where ω_1 and ω_2 are nonzero. If $[\mathbf{a}_{imp}] = 0$, the system becomes a normal sponge layer.

The utility of this combined generation/damping layer is easy to see. By not requiring two separate generation/absorbing layers as with Wei and Kirby (1999) or Chawla and Kirby (2000), space is saved. Undamped free waves are relatively easy to derive compared to the Green's functions in internal wavemakers, or may even be taken from other model outputs. Nonlinear waves, which are a significant issue for internal wave generators using Green's functions, are easy to generate with this new method as long as free wave solutions are known for the system variables. There are only two significant issues to be dealt with: (1) making certain that the generating/absorbing layer is long enough and strong enough to generate and dissipate waves, and (2) making sure that there is no significant re-reflection from free waves entering the sponge layer from the domain. Similar systems have been used with good results in compressible flow computational fluid dynamics to generate and absorb acoustic waves (Bodony, 2006), which are equivalent in many ways to shallow water equations.

Analytic proofs of the system are difficult nonlinearly, but systems are relatively straightforward to analyze for the case of a linear flat bed. However, although nonlinear analytics are difficult, demonstrations of nonlinear accuracy are not, as will be shown. Here, we perform analysis in one horizontal dimension for Boussinesq and shallow water systems although extension to two horizontal dimensions is straightforward. For a linear flat bed with one horizontal dimension, numerous sets of Boussinesq equations and shallow water equations may be represented as (e.g., following Wei and Kirby, 1999), after including the generating/absorbing terms,

$$\begin{aligned} &\eta_{,t} + hU_{,x} + \alpha_1 h^3 U_{,xxx} = \omega_1\left(\eta_{imp} - \eta\right) \\ &U_{,t} + g\eta_{,x} + \alpha h^2 U_{,xxt} = \omega_1\left(U_{imp} - U\right) + \omega_2 \alpha h^2\left(U_{imp,xxt} - U_{,xxt}\right) \end{aligned} \quad (2.2)$$

where g is gravitational acceleration and h is the water depth. To obtain nonlinear shallow water equations, set $\alpha_1 = \alpha = 0$; for Nwogu's (1993) equations, $\alpha_1 = \alpha + 1/3$; to obtain Peregrine's (1967) depth-averaged equations, $\alpha_1 = 0, \alpha = -1/3$.

The undamped ($\omega_1 = \omega_2 = 0$) solution to these equations for free waves traveling in the positive and negative x-directions is

$$\begin{aligned} &\eta_F = \eta_0 \exp[i(kx - \sigma t)] + \eta_1 \exp[i(-kx - \sigma t)] + c.c. \\ &U_F = u_0 \exp[i(kx - \sigma t)] - u_1 \exp[i(-kx - \sigma t)] + c.c. \end{aligned} \quad (2.3)$$

where the radial frequency, σ, is given by

$$\sigma^2 = gk^2 \frac{1 - \alpha_1 (kh)^2}{1 - \alpha (kh)^2} \quad (2.4)$$

and k is the cross-shore wavenumber for a free wave. The relationship between velocities and surface elevations is

$$u_0 = \eta_0 \frac{gk}{\sigma\left(1 - \alpha (kh)^2\right)}. \quad (2.5)$$

Note again that these are also solutions to the damped equations if we take $\eta_{imp} = \eta_F$, and $U_{imp} = U_F$. Thus, the undamped result is a particular solution to the damped equation. To find the full solution to the damped equation, we merely need to find the homogeneous dissipative solution (with $\eta_{imp} = 0, U_{imp} = 0$) and apply boundary conditions based on the problem geometry. The homogeneous solution depends strongly on the spatial variation of the sponge layers,

$$\omega_1(x) = (\widetilde{\omega}_1/L) f(x), \quad \omega_2(x) = (\widetilde{\omega}_2/L) f(x) \quad (2.6)$$

where L is the length of the sponge layer.

Standard sponge layers typically are maximum at the computational boundaries, and have a smooth variation to zero at their furthest extent in the domain. Here, we assume a polynomial variation,

$$f(x) = \begin{matrix} (n+1)\left(\frac{x}{L}\right)^n, 0 \le x \le L, \\ 0, \qquad x < 0 \end{matrix} \quad (2.7)$$

so that the integrated strength of the sponge layers $\int_0^L \omega_i dx = \widetilde{\omega}_i, i = 1, 2$.

Analytical behavior of the generating–absorbing sponge layer is most easily demonstrated using the shallow water equations, as these have straightforward solutions. Details will differ once dispersive terms are added but, as will be shown, the behavior is generally similar although there are some significant differences; e.g., ω_2 has no effect on shallow water equations as there are no mixed space–time terms. In this case, the general solution to this system in the region $0 \le x \le L$ is, for η_{imp} and U_{imp} that satisfy the undamped equations,

$$\begin{aligned} \eta = &\widetilde{\eta}_{impR} \exp(-ikx + i\sigma t) + \widetilde{\eta}_{impL} \exp(ikx + i\sigma t) \\ &+ A_L \exp(ikx + i\sigma t) \exp\left(\frac{\widetilde{\omega}_1}{C_0}\left(\frac{x}{L}\right)^{n+1}\right) \\ &+ A_R \exp(-ikx + i\sigma t) \exp\left(-\frac{\widetilde{\omega}_1}{C_0}\left(\frac{x}{L}\right)^{n+1}\right) + c.c. \end{aligned} \quad (2.8)$$

$$\begin{aligned} U = &\widetilde{\eta}_{impR}\sqrt{g/h} \exp(-ikx + i\sigma t) - \widetilde{\eta}_{impL}\sqrt{g/h} \exp(ikx + i\sigma t) \\ &- A_L\sqrt{g/h} \exp(ikx + i\sigma t) \exp\left(\frac{\widetilde{\omega}_1}{C_0}\left(\frac{x}{L}\right)^{n+1}\right) \\ &+ A_R\sqrt{g/h} \exp(-ikx + i\sigma t) \exp\left(-\frac{\widetilde{\omega}_1}{C_0}\left(\frac{x}{L}\right)^{n+1}\right) + c.c. \end{aligned} \quad (2.9)$$

where $C_0 \equiv (gh_o)^{1/2}$ is the long wave speed.

This system has three parts: (1) undamped left-and-rightward moving imposed waves, (2) leftward-moving damped waves, and (3) rightward-moving damped waves. The general system plus boundary conditions will then give the performance of specific implementations. In the most simple, with $\eta_{imp} = U_{imp} = 0$, a rightward moving damped free wave that enters the sponge layer at $x = 0$ is reflected by a wall boundary at $x = L$, and re-exits moving leftward at $x = 0$ will be damped by a factor of $\exp[-2\widetilde{\omega}_1/C_0]$. Thus, for a damping factor of $\widetilde{\omega}_1/C_0 = 5$, the reflected wave will only be 4.5×10^{-5} times the size of the incoming wave, which is negligible.