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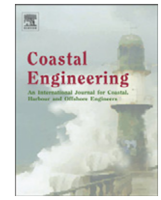

Stokes drift estimation for deep water waves based on short-term variation of wave conditions

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The paper provides a simple analytical method which can be used to give estimates of the Stokes drift based on short-term variation of wave conditions. This is achieved by providing bivariate distributions of wave height and surface Stokes drift as well as wave height and volume Stokes transport for individual random waves within a sea state. The paper presents and discusses statistical aspects of these Stokes drift parameters, as well as examples of results corresponding to typical field conditions.

1. Introduction

The Stokes drift represents an important transport component in the ocean. Locally it is responsible for transport of e.g. contaminated ballast water from ships, oil spills, plankton, and larvae. It is also involved in mixing processes across the interphase between the atmosphere and the ocean. The Stokes drift is obtained as the mean Lagrangian velocity yielding the water particle drift in the wave propagation direction, with its maximum at the surface and decreasing with the depth below the surface. The total mean mass transport is obtained by integrating the Stokes drift over the water depth, also referred to as the volume Stokes transport by Rascle et al. (2008). More details of the Stokes drift are given by e.g. Dean and Dalrymple (1984).

The Stokes drift and the volume Stokes transport were originally defined for regular waves. However, their characteristic quantities for random waves in terms of the sea state parameters significant wave height and characteristic wave periods are also defined (see e.g. Rascle et al. (2008); Webb and Fox-Kemper (2011)). A global database for parameters associated with ocean surface mixing and drift including the surface Stokes drift and the volume Stokes transport among other parameters by performing wave hindcast of the wave parameters was described by Rascle et al. (2008). The hindcast results of Rascle et al. (2008) were improved by Rascle and Ardhuin (2013) using new parameterizations of the physical processes involved (see their 2013 paper and the references therein for more details). Relationships between the wave spectral moments and the Stokes drift in deep water at an arbitrary elevation in the water column were considered by Webb and Fox-Kemper (2011), and inter-comparisons were made using different spectral formulations. Myrhaug (2013, 2014) presented bivariate distributions of significant wave height with surface Stokes drift and volume Stokes transport. Myrhaug (2013) also presented bivariate distributions of spectral peak period with these two Stokes drift parameters. Based on this some statistical aspects of the Stokes drift parameters together with example of results corresponding to typical field conditions were presented.

The purpose of this study is to give a simple analytical method which can be used to estimate the Stokes drift for individual random waves based on short-term variation of wave conditions available in e.g. joint distributions of wave height (H) and wave period (T). This is obtained from parametric models of a joint distribution of wave height and surface Stokes drift as well as a joint distribution of wave height and volume Stokes transport. This is achieved by transformations of the joint distribution of H and T proposed by Longuet-Higgins (1983). Examples of calculating the mean values of the surface Stokes drift and volume Stokes transport within a given sea state corresponding to typical field conditions are also provided to demonstrate the application of the method. Thus, the present results can be used to estimate the Stokes drift for random waves within a sea state based on available wave statistics.

2. Theoretical background

Following Dean and Dalrymple (1984) the mean (time-averaged) Lagrangian mass transport at an elevation z_1 in the water column in finite water depth h is given as

$$\overline{u}_L = \frac{ga^2k^2}{\omega}\frac{\cosh 2k(z_1 + h)}{\sinh 2kh} \tag{1}$$

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Here, g is the acceleration due to gravity, a is the linear wave amplitude, and k is the wave number corresponding to the cyclic wave frequency ω given by the dispersion relationship $\omega^2 = gk \tanh kh$. Eq. (1) indicates that the water particles drift in the wave propagation direction; this drift has its maximum at the mean free surface $z_1 = 0$ and decreases towards the bottom as $z_1 \to -h$. In deep water Eq. (1) reduces to

$$\bar{u}_L = \frac{ga^2k^2}{\omega} e^{2kz_1} \quad ; \quad \omega^2 = gk \tag{2}$$

The Lagrangian mass transport is often referred to as Stokes drift.

The total mean (time- and depth-averaged) mass transport, i.e. obtained by integrating Eq. (1) over the water depth, is given as (Dean and Dalrymple, 1984)

$$M = \frac{\rho g a^2 k}{2\omega} \tag{3}$$

where ρ is the density of the fluid. M is often referred to as the Stokes transport. More details of the Stokes drift and the Stokes transport are given by Dean and Dalrymple (1984).

In deep water (i.e. for large values of kh and $\omega^2 = gk$) and by substituting $a = H/2, \omega = 2\pi/T$, the surface Stokes drift velocity ($z_1 = 0$) in Eq. (1) can be written as

$$U_s = \frac{2\pi^3}{g}\frac{H^2}{T^3} \tag{4}$$

Similarly, in deep water the total mean mass transport in Eq. (3) (also referred to as the volume Stokes transport) can be written as

$$M = \rho \frac{\pi}{4}\frac{H^2}{T} \tag{5}$$

In a sea state of random waves Eqs. (4) and (5) can be taken to represent the surface Stokes drift and the volume Stokes transport, respectively, associated with a single random wave with wave height H and wave period T.

Different models of the joint probability density function (*pdf*) of H and T are given in the literature. Examples are Cavanié et al. (1976), Lindgren and Rychlik (1982), Longuet-Higgins (1983), Myrhaug and Kjeldsen (1984), and Stansell et al. (2004). Comparisons of distributions with observed wave data have been presented by e.g. Srokosz and Challenor (1987) and Myrhaug and Kvålsvold (1995).

In the present paper the Longuet-Higgins (1983) (hereafter referred to as LH83) joint *pdf* of H and T is chosen to serve the purpose of demonstrating how a joint *pdf* of H and T can be used to provide statistics of U_s and M in a sea state of random waves. LH83 was derived by considering the statistics of the wave envelope, that is, the joint distribution of the envelope amplitude and the time derivative of the envelope phase. This distribution is also based on a narrow-band approximation. The LH83 joint *pdf* of wave height and wave period is given as

$$p(h,t) = C\left(\frac{h}{t}\right)^2 \exp\left\{-h^2\left[1+\frac{1}{\nu^2}\left(1-\frac{1}{t}\right)^2\right]\right\} \tag{6}$$

where

$$h = \frac{H}{2\sqrt{2m_0}} \tag{7}$$

$$t = \frac{T}{2\pi\frac{m_0}{m_1}} \tag{8}$$

are the dimensionless wave height and wave period, respectively, and

$$C = \frac{4}{\sqrt{\pi}\nu\left[1+\left(1+\nu^2\right)^{-1/2}\right]} \tag{9}$$

$$\nu^2 = \frac{m_0 m_2}{m_1^2} - 1. \tag{10}$$

Here m_n is the spectral moments defined as

$$m_n = \int_0^\infty \omega^n S(\omega) d\omega; n = 0, 1, 2, ------ \tag{11}$$

where $S(\omega)$ is the single-sided wave spectrum. The parameter ν represents a measure of the bandwidth of the wave spectrum, which may be considered narrow-band if ν is small.

LH83 presented also the marginal *pdf* of h, $p(h)$, and the conditional *pdf* of t given h, $p(t|h)$, given by, respectively

$$p(h) = C\nu\frac{\sqrt{\pi}}{2} h e^{-h^2}\left(1 + erf\left(\frac{h}{\nu}\right)\right) \tag{12}$$

$$p(t|h) = \frac{2}{\sqrt{\pi}\nu(1+erf(h/\nu))}\frac{h}{t^2}\exp\left[-\frac{h^2}{\nu^2}\left(1-\frac{1}{t}\right)^2\right] \tag{13}$$

where $erf(x)$ is the error function defined by

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2} dt \tag{14}$$

Since the spectral moments m_n are constants for each sea state, the *pdfs* in Eqs. (6), (12) and (13) are conditional *pdfs* given in a sea state, i.e. given m_n (or equivalently $S(\omega)$). It should be noted that for a narrow-band process (i.e. $\nu = 0$), Eq. (12) reduces to a Rayleigh *pdf*: $p(h) = 2h \exp(-h^2)$. More details about this joint *pdf* of h and t as ν approaches zero are given in LH83.

By introducing the non-dimensional surface Stokes drift velocity $u = U_s/U_{char}$, Eq. (4) can be re-arranged to

$$u = \frac{h^2}{t^3} \tag{15}$$

where

$$U_{char} = \frac{2m_1^3}{gm_0^2} \tag{16}$$

is a characteristic surface Stokes drift for the sea state. Similarly, by introducing the non-dimensional Stokes transport $m = M/M_{char}$, Eq. (5) can be re-arranged to

$$m = \frac{h^2}{t} \tag{17}$$

where

$$M_{char} = \rho m_1 \tag{18}$$

is a characteristic Stokes transport for the sea state.