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Flow convergence at the tip and edges of a viscous swash front — Experimental and analytical modeling

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ABSTRACT

The details of flow at the tip of a viscous swash front are important to describe the propagation of the wave, the bed shear and to estimate material transport rates and impact forces. This paper presents novel experimental data illustrating the convergence of fluid at swash fronts generated by dam-break flows. Very viscous fluids (detergents) were used to slow the flow sufficiently to enable video tracking of particles on the free surface and within the interior of the flow. The experiments were performed both up a slope and on a horizontal bed. The particle tracking shows that surface particles travel faster than the mean flow, converge on the swash tip and then rapidly decelerate, a process that will induce a high bed shear stress at the swash tip as observed in recent experiments. Particles also converge on the wall boundaries because of the no-slip condition. A simple analytical model is developed to estimate the ratio of the velocity of surface particles and the wave front. For laminar flows, this ratio is found to be 3/2, independent of the bed slope and flow depth, and is in good agreement with the experimental data. The same model approach suggests a ratio of 8/7 for turbulent flows. This flow convergence does not appear to be included in either analytical modeling of the tip region or in basal resistance laws for the swash front and would modify the momentum equation at the swash tip [c.f. Hogg and Pritchard, 2004] and the kinematic boundary condition at the shoreline. The flow convergence is consistent with observations of the behavior and build-up of buoyant debris at the leading edge of tsunami wave front and can be observed in natural swash flows on beaches.

1. Introduction

More accurate descriptions of the flow details at the tip of a swash front are of relevance for improving models for the propagation of waves on beaches and in dam-break flows, for determining the basal resistance in the tip region, and for estimating sediment transport rates and impact forces (Emmett and Moodie, 2008; Hogg and Pritchard, 2004; Othman et al., 2014; Yeh, 2006). Current models, based around the application of an empirical semi-analytical force balance (Hughes, 1995; Puleo and Holland, 2001) or the non-linear shallow water equations assume that the wave-tip region propagates as a solid tip with a uniform flow in the region immediately behind the tip (Chanson, 2006; Whitham, 1955). This assumption also leads to the assumption that the kinematic condition at the wave tip (shoreline) is that fluid particles at the shoreline stay at the shoreline, or equivalently that the velocity of the shoreline and fluid velocity are equal at the shoreline (e.g., Brocchini et al., 2002), which is the most widely adopted shoreline boundary condition for coastal numerical models. The effects of resistance are modeled with a friction coefficient that is applied to the interface between the wave and the bed. The effect of this simplification is that the surface particles propagate at the celerity of the swash tip, as does the momentum. In practice, there is shear in the velocity profile and a boundary layer occurs at the front (Ancey et al., 2009, 2012; Andreini et al., 2012; Hogg and Pritchard, 2004).

Both swash and dam-break wave fronts are one class of a wide range of shallow water flows which are influenced by friction, see Chanson (2006) for a comprehensive review. However, direct measurements of the shear stress at the tip of swash wave fronts do not show good agreement with conventional friction coefficients (Barnes and Baldock, 2010; O'Donoghue et al., 2010); the shear stress within the tip region is particularly high and then decreases very rapidly away from the front. Barnes and Baldock (2010) suggested that this might be because the no-slip condition at the bed leads to flow convergence at the swash tip, which is then overrun by the fluid behind. This mechanism will lead to the constant injection of high momentum fluid into the boundary layer at the swash tip, potentially generating high bed shear stresses.

Prior studies have shown that dam-break velocities increase non-linearly away from the bed (e.g., Ancey et al., 2009, 2012; Andreini et al., 2012; Hogg and Pritchard, 2004). Here we show that this vertical flow structure in a dam break leads to convergence near the leading tip. While a non-uniform velocity profile does not necessarily ensure flow convergence, with hindsight, flow convergence can be readily inferred

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from such prior observations. However, we are unaware of previous experiments that present observations of such flow convergence, or a simple theory to determine the rate of convergence.

This paper considers this issue and presents new experiments that aim to illustrate the details of the flow at the tip of a viscous wave front induced by dam-break swash flows. Inspired by observations of the creeping and rolling motion of lava flows (see e.g., Griffiths, 2000), very viscous fluids (detergents) are used to slow the flow sufficiently to enable video tracking of particles on the free surface and within the interior of the flow. An analytical description of rate of flow convergence at the wave front is developed, and is compared to the measured data, with good agreement. This provides a basis for extrapolation to turbulent flows in water. The paper is organized as follows. Section 2 outlines a new theoretical analysis to predict the rate of flow convergence toward the tip, which is found to be consistent with the viscous flow solution of Huppert (1982). Section 3 presents the details of the experimental setup and particle tracking technique. Results, including photographs and particle trajectories are summarized in Section 4. Final conclusions follow in Section 5.

2. Theory

The present work is concerned with the flow on the free surface of the fluid and in the interior of the flow, rather than the details at the contact line. Consequently, it is not necessary to consider the contact line dynamics for an overall description, consistent with the approach of Huppert (1982). The key assumption is that there is a quasi-steady self-similar flow condition at the swash front, defined in Fig. 1. With this assumption, from continuity, and with discharge per unit width q(x), the tip celerity, c, will be equal to the *mean* flow velocity behind the front, $\overline{u}$:

$$q(x) = \int_0^h u dy = ch = \overline{u}h. \tag{1}$$

Viscous basal drag will result in a non-uniform velocity profile. Taking zero shear stress on the free surface yields a parabolic velocity profile for a laminar flow of fluid with density ρ, and dynamic viscosity, μ:

$$u(y) = \frac{1}{2\mu}\rho g \sin\alpha\left(h^2 - y^2\right) \tag{2}$$

with y measured downward and perpendicular to the free surface. Substitution into Eq. (1) gives the mean velocity

$$\overline{u} = \frac{\rho g \sin\alpha h^2}{3\mu} = c. \tag{3}$$

The velocity of surface particles is obtained from Eq. (2) with $y = 0$. Taking the ratio of the velocity of the surface particles, U_s, to the mean flow velocity or the tip celerity gives:

$$\frac{U_s}{c} = \frac{U_s}{\overline{u}} = \frac{3}{2} \tag{4}$$

which is independent of the bed slope and the flow depth and is a well-known result for uniform free surface laminar flows. The surface curvature can be accounted for by including a correction term of $-\cot\alpha\,\partial h/\partial x$ in Eq. (2), e.g., Ancey et al., 2012; Hogg and Matson, 2009.

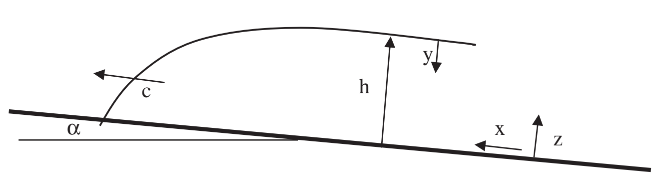

Fig. 1. Definition sketch and the coordinate system for a wave front progressing up slope.

However, on integration, the same term occurs in Eq. (3) and therefore cancels in Eq. (4), giving no change in the rate of convergence. It should be noted that very close to the intersection of the wave tip and the bed the assumption of a shallow flow with negligible vertical component becomes invalid, and the horizontal velocity will reduce compared to the theoretical laminar solution. This can be observed in the data of Andreini et al. (2012). A power law can be used as an alternative (approximation) for parabolic or logarithmic boundary layer profile, which simplifies the algebra in the latter case. Taking z measured perpendicular upward from the bed,

$$\frac{u}{U_s} = \left(\frac{z}{h}\right)^{1/n} \tag{5}$$

yielding

$$\overline{u} = U_s \frac{n}{n+1} \tag{6}$$

and hence

$$U_s = c\frac{n+1}{n}. \tag{7}$$

For $n = 2$, $U_s = 3/2c$, corresponding to laminar flows, and for $n = 7$, $U_s = 8/7c$, corresponding to higher Reynolds number turbulent flows (e.g., Daugherty, 1977). Therefore, while it is well known that surface particles travel faster than the mean flow in steady flows, it is the application of this principle at the swash tip that is relevant here, and which yields a near constant relative velocity between the surface particle and the swash tip, the magnitude of which is controlled by the shear in the velocity profile. Clearly, the relative velocity between the fluid particles and swash tip then depends on the elevation of the fluid within the boundary layer. Basal fluid is left behind the wave front, whereas fluid near the surface converges on the wave front. As a uniform velocity profile is approached, the surface velocity approaches the mean velocity (and the tip celerity), which is the conventional model assumption for the leading edge of swash (Hogg and Pritchard, 2004) and dam-break flows (e.g., Chanson, 2006; Whitham, 1955).

Huppert (1982) provided a laminar solution for the far field swash tip position for the problem of a viscous wave front, in that instance propagating downslope:

$$x_{tip} = Dt^{1/3}, \quad D = \left(\frac{9A^2 g \sin\alpha}{4\upsilon}\right)^{1/3} \tag{8}$$

where A is the initial cross-sectional area of the flow and ν is the kinematic viscosity. The swash tip speed can be derived as:

$$\frac{dx_{tip}}{dt} = \frac{1}{3}Dt^{-2/3}. \tag{9}$$

Huppert (1982) also provides an expression for the depth just behind the swash tip, but not for the surface velocity or the flow profile:

$$h_{tip} = \frac{1.5A}{x_{tip}}. \tag{10}$$

However, Huppert's governing equation is the exact laminar form of the Navier–Stokes equation, so it is assumed here that the surface velocity in that solution is again given by Eq. (2). Combining Eqs. (2), (8), (9) and (10) gives $U_s/c = 3/2$, as before, and independent of slope. Thus, the solutions are consistent and Eqs. (4) and (7) are expected to hold regardless of slope and viscosity. Ancey et al. (2009) provide a more detailed solution for the position of the wave tip and flow depths, including the shape of the surface in the swash tip, which tends quickly to a self-similar shape. Thus the approximation of a quasi-steady self-