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Model predictive control with non-uniformly spaced optimization horizon for multi-timescale processes

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ABSTRACT

Many chemical processes exhibit disparate timescale dynamics with strong coupling between fast, moderate and slow variables. To effectively handle this issue, a model predictive control (MPC) scheme with a non-uniformly spaced optimization horizon is proposed in this paper. This approach implements the time intervals that are small in the near future but large in the distant future, allowing the fast, moderate and slow dynamics to be included in the optimization whilst reducing the number of decision variables. A sufficient condition for ensuring stability for the proposed MPC is developed. The proposed approach is demonstrated using a case study of an industrial paste thickener control problem. While the performance of the proposed approach remains similar to a conventional MPC, it reduces the computational complexity significantly.

1. Introduction

Over the past few decades there have been significant developments in model predictive control (MPC), both in terms of theoretical developments and industrial applications (Qin and Badgwell, 2003). The key idea of MPC is to optimize a control trajectory based on a process model by solving a constrained finite horizon optimization problem (Mayne et al., 2000). It has made a significant impact on the chemical and process industries due to its ability to handle constraints explicitly and deliver high levels of performance (García et al., 1989). In MPC, the length of the optimization horizon (which includes the prediction and control horizons) is to be decided. Although a longer horizon leads to an increase in performance (Geyer, 2011), it is accompanied by a higher computational complexity.

Many chemical processes (e.g., continuous stirred tank reactors (Chang and Aluko, 1984) and biochemical reactors (Bailey and Ollis, 1976)) have dynamics with multiple timescales. Control design for these systems should take into account the presence of timescale multiplicity or it may lead to performance deterioration and closed-loop instability (Christofides and Daoutidis, 1996). For a multi-timescale process, an MPC with a sufficiently long optimization horizon is often required to fully enclose the transients of all dynamics. However, this is not always possible as it may require a small time interval (e.g., Δt_1 in Fig. 3) and a long optimization horizon, resulting in a very complex optimization problem with a large number of decision variables. This has motivated research into approaches to resolve the problem.

The most common approach involves the use of the singular perturbation theory (Kokotović et al., 1999). This method has been widely applied, e.g., in the MPC of nonlinear singularly perturbed systems (Chen et al., 2011, 2012; Ellis et al., 2013). For example, in Chen et al. (2012), the process system is decomposed into two separate reduced-order subsystems evolving in different timescales. Then, a “fast” and a “slow” MPC are designed to regulate the fast and slow dynamics respectively, allowing all dynamics of the process system to be effectively optimized by the MPC controllers. However, this approach requires explicit timescale separation, which may not be the case for many processes, e.g., distillation columns (Lévine and Rouchon, 1991) and the thickening process (Bürger et al., 2004) that exhibit dynamics with a continuum of time constants (with the exception that under several necessary conditions, a system with non-explicit timescale separation may be transformed into a standard singularly perturbed form (Kumar et al., 1998)). In addition, this method requires measurements for both fast and slow variables, which may not always be available, e.g., concentration measurements may not be available in a reactor due to cost limitations (Colantonio et al., 1995). While there are extensions to singular perturbation approaches (Esteban et al., 2013), they are mainly applicable to two-timescale systems.

Another approach is to represent the control trajectories as functions of wavelets and optimize the trajectories by solving

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the wavelet coefficients. This allows the trajectories to contain the dynamics with different timescales (Krishnan and Hoo, 1999). However, when the optimization horizon is long, this approach involves a large number of wavelet coefficients, which increases the computational burden (Wang et al., 2015).

In this paper, we propose a modification to the conventional MPC algorithm for processes exhibiting multi-timescale dynamics. The approach optimizes the control trajectories with a non-uniformly spaced receding optimization horizon – using small time intervals for good accuracy (e.g., Δt_1 in Fig. 3) in the near future, and large time intervals (e.g., Δt_3 in Fig. 3) for reduced computational burden in the distant future. As the proposed approach involves a single controller, it does not require explicit separation of the fast and slow dynamics in the controller design and allows for an arbitrary number of time intervals to be chosen to deal with multi-timescale processes. It can significantly reduce the number of decision variables and thus the computational complexity. It should be pointed out that the proposed approach is different to multi-rate MPC, where different sampling rates are used for sensors and actuators (Scattolini and Schiavoni, 1995).

While most MPC approaches employ uniformly spaced optimization horizons, there are some studies on approaches similar to MPC with non-uniformly spaced optimization horizons. For example, in Goodwin et al. (2006), the idea of non-uniform time quantization was developed for open-loop optimization of mine planning. In Halldorsson et al. (2005), non-equidistant horizons were used to find the open-loop optimal control trajectories, leading to reduced computational burden. Gondhalekar and Imura (2006) studied the effect of varying time intervals to the cost function. However, the resulting control policies often do not guarantee stability, as existing stability results (e.g., based on the terminal constraint (Mayne et al., 2000)) are not directly applicable to the above approaches (Halldorsson et al., 2005). Another way to reduce computational complexity is known as "move blocking" which constrains blocks of adjacent-in-time control action to have the same values (Cagienard et al., 2007; Gondhalekar and Imura, 2010). However, move blocking can be very computationally demanding (at least offline computation) when a long optimization horizon is used. Similar to the non-equidistant horizon based optimization approaches discussed above, the stability of MPC with move blocking is still an open problem (Gondhalekar and Imura, 2010). While the "moving window blocking" approach in Cagienard et al. (2007) ensures stability, it cannot be used to deal with the multi-timescale process dynamics because it requires the blocked inputs to be shifted at each time-step.

In this work, a condition to ensure stability of the closed-loop system of the proposed MPC approach and the process is also derived. The implications of the proposed MPC on the control performance and computational complexity are discussed.

This paper is organized as follows: processes with multi-timescale dynamics are discussed in Section 2 with a motivating example of a paste thickening process used in a coal preparation plant. Section 3 presents the main idea of the proposed MPC algorithm and develops the stability of the approach. Section 4 illustrates the proposed approach with simulation studies, followed by the conclusion in Section 5.

In this paper, the following notations are employed: $\lambda(A)$ denotes the eigenvalues of matrix A. A^T represents the transpose of matrix A. $A \succ 0 (\succeq 0)$ is used to represent a positive (semi-)definite matrix.

2. Processes with multiple timescales dynamics

Many industrial processes have dynamics with multiple timescales, e.g., continuous stirred tank reactors (Chang and Aluko, 1984), biochemical reactors (Bailey and Ollis, 1976), distillation columns (Lévine and Rouchon, 1991), and the thickening process (Bürger et al., 2004). In this section, we illustrate this issue with an example of the thickening process which is widely used in mineral and mining industries, as well as in wastewater treatment plants. It is used to separate water from solids in a slurry. The process involves sedimentation, i.e., settling of tiny particles, often enhanced with flocculant to form larger aggregates. At the same time, overflow which consists of a negligible amount of particulate matter is recycled, thus reducing plant water requirement.

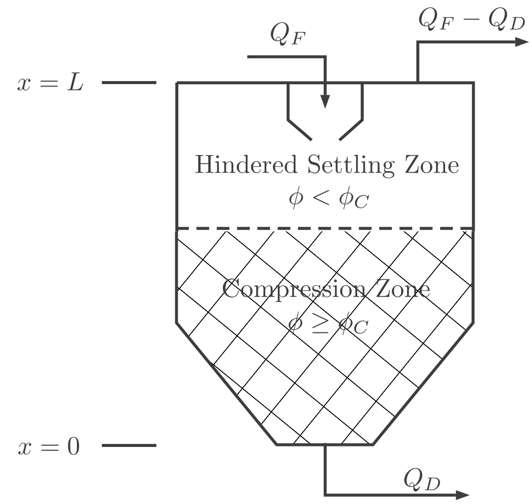

Fig. 1. A continuous operating paste thickener.

Fig. 1 shows a continuous paste thickener fed with a slurry of flow rate Q_F, with the product (thickened slurry) removed at a rate of Q_D from the bottom of the thickener. The overflow from the thickener $Q_F - Q_D$ is recycled back to the plant, and is assumed to contain water only. The critical concentration is ϕ_c, which divides the process into the hindered settling and the compression zones. At this point, also known as the gel point, particles start to coalesce due to close proximity to one another, forming a bed layer which further enhances the dewatering ability.

The sedimentation–consolidation process can be described using the model below (Bürger et al., 2004):

$$\frac{\partial \phi(x,t)}{\partial t} + \frac{1}{S(x)}\frac{\partial}{\partial x}(Q_D(t)\phi + S(x) f_{bk}(\phi)) = \frac{1}{S(x)}\frac{\partial}{\partial x}\left(S(x)\frac{\partial A(\phi)}{\partial x}\right), \tag{1}$$

where ϕ is the solids volume fraction as a function of time (denoted by t) and the height above the base of the paste thickener (denoted by x). The cross sectional area is $S(x)$, $Q_D(t)$ is the underflow volumetric flow rate at the bottom of the thickener, $f_{bk}(\phi)$ is the Kynch batch flux density function, often formulated as (Michaels and Bolger, 1962):

$$f_{bk}(\phi) = \begin{cases} v_\infty \phi \left(1 - \dfrac{\phi}{\phi_{\max}}\right)^N & \text{for} \quad 0 \leq \phi \leq \phi_{\max}, \\ 0 & \text{otherwise}, \end{cases} \tag{2}$$

where v_∞ is the terminal settling velocity of a single particle in an infinite dilution. The parameter N is related to the shape of particles (Moreland, 1963). $A(\phi)$ is the consolidation function, which can be described as:

$$A(\phi) = \int_0^\phi a(s)\,ds, \; a(\phi) = -\frac{f_{bk}(\phi)\sigma_e'(\phi)}{\Delta \rho g \phi}, \tag{3}$$

where $\Delta\rho$ is the difference in solid-liquid density and g is the gravitational acceleration. The solid stress function $\sigma_e(\phi)$, which