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The role of offshore boundary conditions in the uncertainty of numerical prediction of wave overtopping using non-linear shallow water equations

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ABSTRACT

The paper examines the variability of wave overtopping parameters predicted by numerical models based on non-linear shallow water equations, due to the boundary conditions obtained from wave energy density spectra. Free surface elevation time series at the boundary are generated using the principle of linear superposition of the spectral components. The components' phases are assumed to be random, making it possible to generate an infinite number of offshore boundary conditions from only one spectrum.

A reference case was provided by carrying out overtopping tests on a simple concrete structure in a wave flume. Numerical tests using the measured free surface elevation at the toe of the structure were carried out. Three parameters were analysed throughout the paper: the overtopping discharge, the probability of overtopping and the maximum overtopping volume. These showed very good agreement between the numerical solver prediction and the overtopping measurements. Subsequently, the measured spectra at the toe were used to generate a population of reconstructed offshore boundary time series for each test, following a Monte Carlo approach. A sensitivity analysis determined that 500 tests were suitable to perform a statistical analysis on the predicted overtopping parameters. Results of these tests show that the variability in the predicted parameters is higher for the smaller number of overtopping waves in the modelled range and decreases significantly as overtopping becomes more frequent. The characteristics of the distributions of the predictions have been studied. The average value of the three parameters has been compared with the measurements. Although the accuracy is lower than that achieved by the model when the measured time series are used at the boundary, the prediction is still fairly accurate above all for the highest overtopping discharges. The distribution of the modelled probability of overtopping was found to follow a normal distribution, while the maximum value follows a GEV one. The overtopping discharge shows a more complex behaviour, values in the middle of the tested range follow a Weibull distribution, while a normal distribution describes the top end of the range better.

Results indicate that when the probability of overtopping is smaller than 5%, a sensitivity analysis on the seeding of the offshore boundary conditions is recommended.

1. Introduction

The prediction of wave overtopping is one of the most important steps in the assessment of the hydraulic performance of coastal defence structures. This can be achieved by using three different approaches: by empirical formulae available in literature, by physical modelling using laboratory tests or by numerical simulation of the hydraulic response of the structure. Extensive research has been carried out in the development of suitable methodologies following these three approaches.

The empirical approach and the physical modelling have been comprehensively developed and reliable tools are available.

Empirical formulas have been the first tools to be developed and used by engineers. Recently an artificial neural network (ANN) has been developed during the EU-programme CLASH (Crest Level Assessment of Coastal Structures by full scale monitoring, neural network prediction and Hazard analysis on permissible wave overtopping). The ANN has been trained using a large database of laboratory tests and is able to take into account complex structural geometries, making it the most comprehensive and accurate empirical predictive tool to date. Pullen et al. (2007) provide guidelines on the applications of these predictive tools.

Physical modelling is considered to be a reliable approach to predict overtopping at coastal structures, above all when complex

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layouts and wave conditions are considered. During CLASH, model and scale effects have been analysed (see Franco et al., 2009; Geeraerts et al., 2009).

Numerical modelling is also frequently used to predict wave overtopping. Phase resolving models are particularly suitable as they can simulate individual overtopping events. Within this class of models, depth integrated ones are very popular, when sloping structures are considered, due to their simplicity and low computational requirements. A number of models based on the non-linear shallow water equations (NLSWE) have been proposed for wave overtopping prediction. Kobayashi and Wurjanto (1989) proposed a NLSWE solver based on a finite difference Lax–Wendroff scheme. Subsequently, solvers using finite volumes became more popular since they are more efficient with wet and dry interfaces; Dodd (1998) proposed a finite volume scheme involving a Roe-type Riemann solver and tested it using laboratory data. More recently, Godunov-type schemes became very popular in NLSWE solvers since they allow a straightforward formulation of the shoreline boundary conditions. Two examples are given by Hu et al. (2000) and Briganti and Dodd (2009a). Hu et al. (2000) tested the model on sloping seawalls demonstrating that NLSWE can be used also for steep slopes. This capability was further confirmed in Shiach et al. (2004). This study used the same model of Hu et al. (2000) against laboratory experiments of overtopping on a very steep wall. For the less impactive waves tested the model performed satisfactorily. The authors also highlighted the importance of offshore boundary conditions that should be prescribed close to the structure to guarantee accuracy.

Briganti and Dodd (2009a) is the only work on a NLSWE solver that simulates numerically the analytical solution for overtopping, i.e. the one by Peregrine and Williams (2001).

Recently, Boussinesq type models have been used extensively for wave overtopping research. Their advantage with respect to NLSWE is to model waves in intermediate water depth conditions. This makes it possible to locate the offshore boundary further from the structure than NLSWE solvers allow and to control the onset of wave breaking. In this way the propagation of the waves on the foreshore and the breaking process is better described. Examples of this class of models are given by Stansby (2003), Lynett et al. (2010), McCabe et al. (2013) and Tonelli and Petti (2013) among others.

Lynett et al. (2010) compared the prediction of overtopping from different types of models using a test case involving a sloping structure and regular waves generated at the toe of the structure, also used in Dodd (1998). The authors concluded that all the models tested show a similar level of accuracy in simulating the chosen experiment.

When phase resolving models are used, a free surface elevation and velocity time series are required at the seaward boundary. In design applications, these might not be available. Frequently the modeller is provided with the incident energy density spectra retrieved by a wave buoy or computed by a large scale spectral model such as SWAN (Booij et al., 1999). A time series will then be reconstructed from these spectra to be used as boundary conditions. Since the energy density spectra provide only information on the amplitude of the components, it is usually assumed that the phases of these components are randomly distributed. To create the randomly generated phases, an initial seed value is required to generate a population of uniformly distributed random phases. By varying this value for each simulation a different time series will be produced.

There are two issues related to this procedure. First, this assumption implies a linear superposition of wave components that is strictly valid only for linear waves and it is often considered valid in deep water. Notwithstanding this, the approximation is used in many existing intermediate and shallow water models proposed in the literature. Recent examples of such an assumption in the computation of run-up and overtopping are given in Zijlema et al. (2011), Shi et al. (2012), McCabe et al. (2013) and McCabe et al. (2011).

Secondly, from every energy density spectra an infinite number of different wave series can be generated by changing the seeding of the random phase distribution. Evidence that this process plays an important role in the variability of the results is given in McCabe et al. (2011) for the run-up prediction and McCabe et al. (2013) for overtopping. The authors compared different run-up heights and overtopping volumes resulting from different free surface time series at the boundary, all obtained by the same spectra, hence with the same energy. The results showed that the parameters under study significantly vary with the random seeding used. It has to be noted that the physical phenomenon of overtopping itself is influenced by the specific sequence of waves that arrives at the structure. This aspect has been recognised in early studies of wave overtopping (see Stsuruta and Goda, 1968) but it has not been investigated experimentally in depth until recently. Hunt-Raby et al. (2011) analysed the role of the preceding wave on the individual wave overtopping volume and the role of wave shape using focused wave groups. They found that the shape of the extreme wave in the group and the presence of a deep trough both affect the individual wave overtopping.

The uncertainty in the overtopping prediction introduced by the coupling with spectral models or data is under-studied. In other fields, such as river flood numerical modelling, the study of uncertainty is well established (e.g. Hall, 2003; Hall and Solomatine, 2008; Hall et al., 2009 among others). Pullen et al. (2007) describes the uncertainty in empirical formulas and laboratory experiments, where several studies on this topic, such as Pearson et al. (2001) have been carried out. However, the lack of specific study on the uncertainty in numerical models is one of the reasons why, in Pullen et al. (2007) this approach is not considered as reliable as the aforementioned two approaches.

Using the same classification adopted in Hall (2003), the reconstruction of offshore boundary conditions introduces an epistemic (or model) uncertainty and the present study will quantify it using a simple test case.

The paper is organised as follows. The following section will introduce and describe the experiments carried out in order to obtain a reference case for the numerical model. Section 3 will briefly describe the numerical model and the validation of the model using experimental data and Section 4 will describe the Monte Carlo method used to analyse the effect of the reconstruction of the boundary time series from a single energy density spectrum. Section 5 will examine the variability of the results of the numerical experiments. Section 6 will discuss the key points of the results; and finally Section 7 will present the overall conclusions of this research.

2. Laboratory tests

2.1. Introduction

The study of the variability of model predictions required a reference test case in which the hydraulic input and output conditions were known. This test case was provided by simulating wave overtopping in a series of small scale laboratory tests carried out in a two dimensional (2D) wave flume at HR Wallingford.

The structure tested was a simple concrete impermeable slope. This configuration was chosen in order to be able to neglect the permeability of the armour layer, which would introduce a further modelling uncertainty. Wave conditions were chosen to be suitably modelled with a NLSWE solver. Among the wave conditions tested the analysis focused on those that produced levels of overtopping that are comparable to the limits for pedestrians defined by Pullen et al. (2007).