Coastal Engineering 58 (2011) 877–889

Contents lists available at ScienceDirect

Coastal Engineering

journal homepage: www.elsevier.com/locate/coastaleng

A new predictive formula for inception of regular wave breaking

Yu Liu, Xiaojing Niu, Xiping Yu *

State Key Laboratory of Hydroscience and Engineering, Department of Hydraulic Engineering, Tsinghua University, Beijing, China

ARTICLE INFO

Article history:
Received 15 December 2010
Received in revised form 29 April 2011
Accepted 2 May 2011
Available online 31 May 2011

Keywords:
Wave breaking
Empirical formula
Collected data
Fitting
Verification

ABSTRACT

Efforts are made to enhance the predictive formula for the inception of wave breaking. To achieve success, the existing formulas are extensively reviewed. They are categorized into four types, i.e., the McCowan type, the Miche type, the Goda type and the Munk type. The inherent relations among the different types are then exploited. The differences among each formula within a group are also discussed. Four representative formulas from the different types are chosen to compare with the measured data for a total number of 1193 cases reported in literatures. It is shown that Goda's and Ostendorf and Madsen's formulas are advantageous in general among the selected ones. Goda's formula, however, is found to be inaccurate as the beach slope becomes steeper than 1/10. Ostendorf and Madsen's formula is fairly good even for cases of very steep slopes, but its accuracy for the cases of ordinary slopes is not as good as Goda's. A new predictive formula for the inception of wave breaking is proposed. The unique index, defined by $\psi'_b = (1.21 - 3.30\lambda_b)(1.48 - 0.54\gamma_b)\psi_b$, where $\psi_b = gH_b/C_b^2$, H_b is the breaking wave height, C_b is the breaking wave celerity, λ_b is the breaking wave steepness, γ_b is the relative breaking wave height, and g is the gravity acceleration, is introduced. The incipient condition of wave breaking is then given by $\psi'_b = 0.69$. This formula is a significant improvement to the existing ones in terms of the accuracy. In addition, it is a local relation. Further verification shows that the proposed formula performs similarly well when applied to the field and to the waves over permeable bed.

1. Introduction

Wave breaking is an amazing phenomenon to various people. The artists are amazed at its dynamic beauty and the scientists are amazed at its beautiful dynamics. The feeling of the coastal engineers on wave breaking, however, is often mixed. Breaking wave dynamics is a theoretically unsolved topic, and the breaking processes are so complicated that an observation can provide only limited information with generality even by means of the most recent high technology. On the other hand, the practicing engineers often have to provide solutions anyhow to the phenomenon because it is so closely related to the important problems such as the wave force on structures and the wave induced sediment transport in the surf zone.

It is believed that a wave break as an intrinsic relation is satisfied among the breaker height, the local water depth, the local wavelength, the bottom slope, and probably also some other parameters. To find this relation in a general form, however, is not easy. Efforts have been continued for more than a century. The initial contributions now are usually attributed to Michell (1893) and McCowan (1894). Based on the assumption that a solitary wave breaks as its crest angle approaches a limiting value or the fluid velocity at the crest surpasses the celerity of the profile, McCowan (1894) derived the following relation:

$$\frac{H_b}{h_b} = 0.78 \quad (1)$$

where H_b is the breaker height, h_b is the water depth at the breaking point. Michell (1893) found the limiting steepness of deepwater waves or the breaking condition of deepwater waves:

$$\frac{H_b}{L_b} = 0.142 \quad (2)$$

where L_b is the breaking wavelength. Miche (1944) then generalized Michell's (1893) condition and obtained the distinguished formula for periodic waves over arbitrary water depth:

$$\frac{H_b}{L_b} = 0.142 \tanh \frac{2\pi h_b}{L_b} \quad (3)$$

In engineering applications, Goda's (1970, 1975) formula seems to have gained the highest reputation. After a slight modification to enhance its performance for steep slopes (Goda, 2010), Goda's formula can be written as

$$\frac{H_b}{L_0} = 0.17\left\{1 - \exp\left[-1.5\frac{\pi h_b}{L_0}\left(1 + 11s^{4/3}\right)\right]\right\} \quad (4)$$

where s is the beach slope.

* Corresponding author. Tel.: +86 10 62776777.
E-mail address: yuxiping@tsinghua.edu.cn (X. Yu).

0378-3839/$ – see front matter
doi:10.1016/j.coastaleng.2011.05.004

Table 1
McCowan type formulas for inception of wave breaking.

Functional form	Sources
0.73	Boussinesq (1871)
	Laitone (1960)
0.78	McCowan (1894)
0.83	Gwyther (1900)
	Davies (1952)
	Yamada et al. (1968)
	Yamada (1957)
	Lenau (1966)
	Longuet-Higgins and Fenton (1974)
	Witting (1975)
	Longuet-Higgins and Fox (1977)
0.87	Chappelear (1959)
1.03	Packham (1952)
$[1.40-\max(s,\ 0.07)]^{-1}$	Galvin (1969)
$0.72(1+6.4s)$	Madsen (1976)
$1.062+0.137\log(s\lambda_0^{-1/2})$	Battjes (1974)
$1.1s^{1/6}\lambda_0^{-1/12}$	Sunamura (1980)
$0.937s^{0.155}\lambda_0^{-0.13}$	Singamsetti and Wind (1980)
$1.14s^{0.21}\lambda_0^{-0.105}$	Larson and Kraus (1989)
$1.12(1+e^{-60s})^{-1}-5.0(1-e^{-43s})\lambda_0$	Smith and Kraus (1990)
$0.284\lambda_0^{-1/2}\tanh[\pi\lambda_0^{1/2}]$	Camenen and Larson (2007)

It is of interest to note that a tremendous number of formulas have been proposed to describe the incipient condition of wave breaking up to now. The earlier studies were mainly focusing on the solitary waves and the waves under deep water conditions. The periodic waves at the breaking point were usually approximated as solitary waves at that time. After the 1940s, the breaking of periodic waves on sloping beaches has been emphasized, and rather general formulas for the inception of breaking then became available. Detailed reviews of the existing researches have been made by Galvin (1972), Sawaragi (1973), Rattanapitikon et al. (2003), Camenen and Larson (2007), and Goda (2010). Difficulties in the establishment of a universal formula are probably that too many factors affect wave breaking. The inherent variability of the phenomena (Goda, 2010) further complicates the problem. In spite of this fact, the efforts to obtain a "better" formula have never stopped during the past decades, probably because a generally valid formula is of too much keen interest to scientists and engineers.

A reliable formula or diagram for the inception of wave breaking may have to be fitted or verified by a significantly large number of data covering a wide range of beach topography and wave conditions. Weggel's (1972) formula as well as Komar and Gaughan's (1972) formula, both recommended by Coastal Engineering Manual authorized by Coastal and Hydraulics Laboratory, US Army Corps of Engineers, are certainly based on a reasonably large number of laboratory data. Goda's (1970, 1975) formula, which has been considered as the standard in the Asian coastal engineering community and also highly appreciated by the coastal engineers from other countries, fitted 215 sets of data from 8 different sources and covered a wide range of the bottom slope from 1/100 to 1/9. Other achievements that have been frequently mentioned in literatures or otherwise are equivalently reliable, including those of Iversen (1951), Ostendorf and Madsen (1979), Singamsetti and Wind (1980), Larson and Kraus (1989), Smith and Kraus (1990), Rattanapitikon and Shibayama (2000), Camenen and Larson (2007), are all based on a large number of data measured by the authors themselves, or by collecting from various sources, or by a combination of both ways.

Table 2
Miche type formulas for inception of wave breaking.

α	ξ	Sources
0.142	1.0	Miche (1944)
0.14	0.9	Battjes and Janssen (1978)
0.14	$0.8+5.0\min(s,0.1)$	Ostendorf and Madsen (1979)
0.14	$0.57+0.45\tanh(33\lambda_0)$	Battjes and Stive (1985)
$0.127e^{4s}$	1.0	Kamphuis (1991)
0.14	$-11.21s^2+5.01s+0.91$	Rattanapitikon and Shibayama (2000)

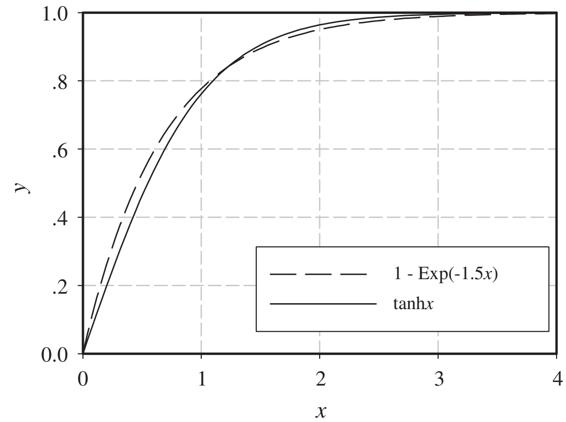

Fig. 1. Comparison of tanh x and $1-\mathrm{Exp}(-1.5x)$.

Critical investigations into the accuracy of the existing formulas for the inception of wave breaking by comparing them with a large number of measured data have also been carried out during the past two decades. Kamphuis (1991) used 225 sets of data obtained by himself to investigate the accuracy of 11 formulas; Rattanapitikon and Shibayama (2000) collected 574 sets of data from 24 sources to examine 24 formulas; Rattanapitikon et al. (2003) further solidified the conclusions of Rattanapitikon and Shibayama (2000) by adding another 121 sets of data measured in large scale wave flumes; Camenen and Larson (2007) also collected more than 500 sets of data from 22 published sources covering a wide range of beach slopes and wave conditions to compare the accuracy of 6 existing formulas.

The present study is trying to make an even more critical comparison of the formulas that are most widely preferred by coastal engineers and are considered as accurate enough within the inherent variability of the phenomena, by increasing the number of data for verification. At the same time, we propose a new formula with an expectation that it can fit the experimental data with evidently smaller errors, and then verify the formula under some critical conditions.

2. Inherence of existing formulas

The breaking condition for the solitary waves over a constant water depth can be expressed by Eq. (1) as many authors pointed out. Different authors, however, obtained different values for the constant. If the formula is generalized to represent the breaking condition for periodic waves on beaches, the constant may then have to be replaced by a function of the beach slope and probably also the deep water wave conditions. Thus, Eq. (1) has a general form:

$$\frac{H_b}{h_b}=\gamma(s,\lambda_0) \tag{5}$$

where $\lambda_0=H_0/L_0$ is the deep water wave steepness with H_0 being the incident wave height. Eq. (5) is called the McCowan (1894) type formula for the inception of wave breaking in the present study. The

Table 3
Goda type formulas for inception of wave breaking.

α'	ξ'	Sources
0.17	$0.5+7.5s^{4/3}$	Goda (1970)
0.17	$0.52+2.36s-5.40s^2$	Rattanapitikon and Shibayama (2000)
0.17	$0.5+5.5s^{4/3}$	Goda (2010)