Coastal Engineering 56 (2009) 479–483

Contents lists available at ScienceDirect

Coastal Engineering

journal homepage: www.elsevier.com/locate/coastaleng

Characteristics of developing waves as a function of atmospheric conditions, water properties, fetch and duration

J.P. Le Roux *

Departamento de Geología, Facultad de Ciencias Físicas y Matemáticas, Universidad de Chile, Casilla 13518, Correo 21, Santiago, Chile

ARTICLE INFO

Article history:
Received 18 December 2007
Received in revised form 1 September 2008
Accepted 28 October 2008
Available online 2 December 2008

Keywords:
Developing waves
Wave period
Wave height
Wavelength
Fetch
Wind duration

ABSTRACT

For any specific wind speed, waves grow in period, height and length as a function of the wind duration and fetch until maximum values are reached, at which point the waves are considered to be fully developed. Although equations and nomograms exist to predict the parameters of developing waves for shorter fetch or duration conditions at different wind speeds, these either do not incorporate important variables such as the air and water temperature, or do not consider the combined effect of fetch and duration. Here, the wind conditions required for a fully developed sea are calculated from maximum wave heights as determined from the wind speed, together with a published growth law based on the friction velocity. This allows the parameters of developing waves to be estimated for any combination of wind velocity, fetch and duration, while also taking account of atmospheric conditions and water properties.

1. Introduction

Estimation of long-term wave conditions is of vital importance in ocean and coastal engineering, whereas the forecast of short-term conditions is critical for maritime activities. While fully developed sea conditions (*FDS*) are often assumed in long-term forecasts, such a state is not necessarily reached in enclosed or semi-enclosed water bodies such as lakes or bays, where a short fetch will limit wave development. This is particularly true of storms, because higher wind speeds require a longer fetch to produce *FDS* waves. For short-term forecasts, the wind duration also plays an important role both on the open sea and in smaller water bodies. Developing waves may be more hazardous than in their fully developed state, because in spite of being lower they are normally much steeper, which poses a danger especially for smaller vessels.

Although much work has been done on wave growth as a function of wind speed, fetch or duration (e.g. Inoue, 1967; Barnett, 1968; Bunting, 1970; Hasselmann et al., 1973; Toba, 1978; Forristall et al., 1978; Resio and Vincent, 1979; Resio, 1981, 1987, 1988; Kahma, 1981; Kitaigorodskii, 1983; Hasselmann et al., 1985; Resio and Perrie, 1989; Cardone, 1992; Van Vledder and Holthuisjen, 1993; Demirbilek et al., 1993), a simple model taking account of all three variables at the same time has been lacking. In this paper, previous work on fully developed waves (Le Roux, 2007a,b, in press) is used together with a growth law based on the JONSWAP (Joint North Sea Wave Project) spectrum (Hasselmann et al., 1973; Demirbilek et al., 1993; Resio et al., 2003) to develop such a method.

2. Atmospheric conditions and water properties

The characteristics of gravity waves are not only a function of the wind conditions, but also depend on the physical properties of the water and air, in particular the difference between the water and air temperature (Geernaert et al., 1986). This difference (Δ°C) has a direct influence on the drag force generated by wind friction on the water surface, so that it is necessary to determine the drag for different atmospheric conditions. The wind friction velocity U_{a*} (the subscripts $_a$ and $_w$ referring to air and water, respectively) is estimated by Demirbilek et al. (1993) as follows:

$$U_{a*} = \sqrt{\left(C_{da} U_a^2\right)} \tag{1}$$

where the dimensionless wind drag coefficient C_{da} is given by

$$C_{da} = 0.001(1.1 + 0.035 U_a). \tag{2}$$

Eq. (2) gives an approximation of the drag coefficient for "normal" weather conditions, but does not take the air–water temperature difference ($\Delta°C = °C_a - °C_w$) into account. A more rigorous solution is given by a 3-D graph in Resio et al. (2003, Fig. II-2-5) that plots C_{da} as a series of curves against U_a for different values of Δ°C (Geernaert et al., 1986; Smith, 1988). This graph can be recast into the single equation:

$$C_{da} = \left(-1.7\times10^{-8}\Delta°C^3 - 1.4\times10^{-6}\Delta°C^2 - 3\times10^{-5}\Delta°C + 0.001\right) \exp\left[U_a\left(-1.6\times10^{-6}\Delta°C^3 + 2\times10^{-5}\Delta°C^2 + 0.001\Delta°C + 0.0324\right)\right]. \tag{3}$$

At a wind speed of 10 m s^{-1}, an atmospheric pressure of 1010 mb, a relative humidity of 80%, and air and water temperatures of 20 and

* Tel.: +56 2 9784123.
E-mail address: jroux@cec.uchile.cl.

0378-3839/$ – see front matter
doi:10.1016/j.coastaleng.2008.10.007

23 °C, respectively, the values from Eqs. (2) and (3) inserted into Eq. (1) coincide at 0.3808 m s^{-1}. For practical reasons, this is taken here as the "normal condition" or *NC*.

Another factor probably playing a role in wave formation is the water and air density, as discussed below. The density of seawater depends on its temperature and salinity. For a normal seawater salinity of 35‰

$$\rho_w = 1000 + \left(-0.0051°C_w^2 - 0.064°C_w + 28.109\right)\ \mathrm{kg\ m^{-3}} \quad (4)$$

which yields 1023.9391 kg m^{-3} for sea water at 23 °C.

Air density ρ_a, on the other hand, is a function of its temperature and relative humidity Γ, as well as the isobaric pressure P_a, vapor pressure P_v and saturated vapor pressure P_{vs} in millibar (mb).

$$\rho_a = 1000\{P_a/[2870.5(273.15+°C_a)] - P_v/[4614.95(273.15+°C_a)]\}\ \mathrm{kg\ m^{-3}} \quad (5)$$

where

$$P_v = (\Gamma/100)P_{vs} \text{ and } P_{vs} = 6.1078\times10^{7.5°C_a/(237.3+°C_a)}. \quad (6)$$

For the *NC*, ρ_a works out at 1.18643 kg m^{-3}.

3. Wave height as related to wind speed and the air/water density ratio

Air density probably plays a role in wave height in that the wind energy (E_a) would increase with a higher ρ_a given the same wind speed U_a, because the total energy of any moving fluid according to the Bernoulli equation is given by

$$E_a = 0.5\rho_a U_a^2 + \rho_a g h + P_a \quad (7)$$

where the first and second terms in Eq. (7) refer to the kinetic and potential energies, respectively, and h is a length term. Both the kinematic and potential energies of the wind must therefore increase with the air density, whereas Eq. (5) shows that P_a is also directly proportional to ρ_a.

The total wave energy E_w per unit length of wave crest, on the other hand, is the sum of the kinetic energy and potential energy given by (Demirbilek and Vincent, 2002)

$$E_w = \rho_w g H_o^2 L_o/16 + \rho_w g H_o^2 L_o/16 = \rho_w g H_o^2 L_o/8 \quad (8)$$

where H is the wave height and L the wavelength, the subscript $_o$ referring to the deepwater condition. The potential energy results from that part of the fluid mass being above the still water level (*SWL*), thus being directly related to the wave height, whereas the kinetic energy is due to water particle velocities associated with wave motion (Demirbilek and Vincent, 2002). Eq. (8) shows that the total wave energy increases with the wave height and length as well as the water density. It follows that, in the case of wind energy being transferred to the waves, more energy would be required to reach the same wavelength and height in water with a higher density than the other way round. Considering Eq. (7), the wave height should therefore be directly proportional to the ratio $\rho_a U_a^2/\rho_w g$ (the gravity acceleration being added to maintain the dimensional correctness of this ratio).

Le Roux (in press), based on Demirbilek et al. (1993) and Resio et al. (2003), showed that the fully developed deepwater wave height is given by

$$H_o = gT_w^2/18\pi^2 \quad (9)$$

where T_w is the fully developed wave period obtained from

$$T_w = 2\pi U_a/g. \quad (10)$$

Setting $C_1\rho_a U_a^2/\rho_w g = gT_w^2/18\pi^2$ (where C_1 is a dimensionless proportionality constant) and replacing T_w by $2\pi U_a/g$, this yields

$$H_o = 2C_1\rho_a U_a^2/9g\rho_w. \quad (11)$$

For the "normal condition" or *NC*, C_1 has a value of 863.042. Rearranged, $C_1(\rho_a/\rho_w) = 9gH_o/2U_a^2 = 1$, where the second term is an inverted wave Froude number.

Table 1 compares the values of H_o and T_w as obtained from Eqs. (11) and (10) with the nomograms of Resio et al. (2003) and an equation of Demirbilek et al. (1993). T_w as calculated here corresponds very well with the average between the nomogram values of Resio et al. (2003) and the equation of Demirbilek et al. (1993), whereas H_o agrees exactly with the equation of Demirbilek et al. (1993), but somewhat underestimates the nomogram values of Resio et al. (2003).

4. Fetch and duration as a function of the fully developed wave height

Waves grow in height and length not only in relation to the velocity U_a of the wind, but also to its duration T_a and fetch F, the latter being defined as the distance that the wind blows over open water without a significant change in direction (<15°) or sustained speed (<2.5 m s^{-1}).

Demirbilek et al. (1993) proposed an equation modeling wave growth with fetch based on the JONSWAP data:

$$gH_{FL}/U_{a*}^2 = 0.0413\left(gF/U_{a*}^2\right)^{1/2} \quad (12)$$

where H_{FL} is a fetch-limited, energy-based significant wave height.

Table 1
Comparison of wave heights and periods under fully developed sea conditions at different wind velocities as predicted Eqs. (9) and (11), nomograms in Resio et al. (2003) (DBT), and Eqs. (12) of Demirbilek et al. (1993) (DBT) and (14) of Resio et al. (2003) (RBT)

U_a m s^{-1}	U_{a*} m s^{-1}	T_w s This paper	T_w s Nomogram RBT	T_w s Eq. (18) DBT	H_o m This paper	H_o m Eq. (12) DBT	H_o m Nomogram RBT	F_{FDS} m This paper	T_{aFDS} s This paper
2.5	0.0851	1.6	1.6	1.6	0.14	0.14	0.14	15,566	21,974
5	0.1768	3.2	3.3	3.1	0.57	0.57	0.60	59,780	42,233
7.5	0.2756	4.8	5.1	4.6	1.27	1.27	1.35	122,129	58,645
10	0.3808	6.4	6.6	6.1	2.27	2.27	2.45	204,375	74,216
12.5	0.4937	8.0	8.4	7.5	3.54	3.54	4.13	295,699	87,067
15	0.6148	9.6	11.1	8.9	5.10	5.10	6.14	395,769	98,286
17.5	0.7445	11.2	11.8	10.2	6.94	6.94	8.25	499,756	107,727
20	0.8832	12.8	13.0	11.5	9.06	9.06	11.51	605,212	115,619

U_{a*} was calculated for both sets of equations using Eqs. (1), (3) and (4)–(6) with water and air temperatures of 23 and 20 °C, respectively, a water salinity of 35%, a relative air humidity of 80% and a barometric pressure of 1010 mb.