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Mixed-integer bilevel optimization for capacity planning with rational markets

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ABSTRACT

We formulate the capacity expansion planning as a bilevel optimization to model the hierarchical decision structure involving industrial producers and consumers. The formulation is a mixed-integer bilevel linear program in which the upper level maximizes the profit of a producer and the lower level minimizes the cost paid by markets. The upper-level problem includes mixed-integer variables that establish the expansion plan; the lower level problem is an LP that decides demands assignments. We reformulate the bilevel optimization as a single-level problem using two different approaches: KKT reformulation and duality-based reformulation. We analyze the performance of these reformulations and compare their results with the expansion plans obtained from the traditional single-level formulation. For the solution of large-scale problems, we propose improvements on the duality-based reformulation that allows reducing the number of variables and constraints. The formulations and the solution methods are illustrated with examples from the air separation industry.

1. Introduction

Capacity expansion is one of the most important strategic decisions for industrial gas companies. In this industry, most of the markets are served by local producers because of the competitive advantage given by the location of production facilities. The dynamics of the industrial gas markets imply that companies must anticipate demand increases in order to plan their capacity expansion, maintain supply availability, and avoid regional incursion of new producers. The selection of the right investment and distribution plan plays a critical role for companies in this environment. A rigorous approach based on mathematical modeling and optimization offers the possibility to find the investment and distribution plan that yields the greatest economic benefit.

A rather large body of literature has been published on capacity planning problems in several industries (Luss, 1982). Since the late 1950s, capacity expansion planning has been studied to develop models and solution approaches for diverse applications in the process industries (Sahinidis and Grossmann, 1992), communication networks (Chang and Gavish, 1993), electric power services (Murphy et al., 1982), and water resource systems (Nainis and Haimes, 1975). Sahinidis et al. (1989) proposed a comprehensive MILP model for long range planning of process networks. Van den Heever and Grossmann (1999) used disjunctive programming to extend this methodology to multi-period design and planning of nonlinear chemical processes. An MILP formulation that integrates scheduling with capacity planning for product development was presented by Maravelias and Grossmann (2001). Sundaramoorthy et al. (2012) proposed a two-stage stochastic programming formulation for the integration of capacity and operations planning. In summary, capacity planning is considered a central problem for enterprise-wide optimization, a topic for which comprehensive reviews are available (Grossmann, 2005, 2012).

Despite the importance of capacity expansion in industry, the study of the problem in a competitive environment has not received much attention. Soyster and Murphy (1989) formulated a capacity planning problem for a perfectly competitive market. However, perfect competition is a strong assumption. A more realistic hypothesis is to assume an oligopolistic market as presented by Murphy and Smeers (2005). Game theory models have also been

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used (Zamarripa et al., 2012) for supply chain planning in cooperative and competitive environments.

The competition between two players whose decisions are made sequentially can be modeled as a Stackelberg game (von Stackelberg, 2011). A Stackelberg competition is an extensive game with perfect information in which the leader chooses his actions before the follower has the opportunity to play. It is known that the most interesting equilibria of such games correspond to the solution of a bilevel optimization problem (Osborne and Rubenstein, 1994).

Bilevel optimization problems are mathematical programs with optimization problems in the constraints (Bracken and McGill, 1973). They are suitable to model problems in which two independent decision makers try to optimize their own objective functions (Candler and Townsley, 1982; Bard and Moore, 1992). We present a mixed-integer linear bilevel formulation for the capacity planning of an industrial gas company operating in a competitive environment. The purpose of the upper-level problem is to determine the investment and distribution plan that maximizes the Net Present Value (NPV). The response of markets that can choose among different producers is modeled in the lower-level as a Linear Programming (LP) problem. The lower-level objective function is selected to represent the rational behavior of the markets.

Solution approaches for bilevel optimization problems with lower-level LPs leverage the fact that optimal solutions occur at vertices of the region described by upper and lower level constraints. They rely on vertex enumeration, directional derivatives, penalty terms, or optimality conditions (Saharidis et al., 2013). The most direct approach is to reformulate the bilevel optimization as a single-level problem using the optimality conditions of the lower-level LP. The classic reformulation using Karush–Kuhn–Tucker (KKT) conditions maintains linearity of the problem except for the introduction of complementarity constraints (Fortuny-Amat and McCarl, 1981; Bard and Falk, 1982; Bialas and Karwan, 1982). An equivalent reformulation replaces the lower level problem by its primal and dual constraints, and guarantees optimality by enforcing strong duality (Motto et al., 2005; Garces et al., 2009).

Strategic investment planning for electric power networks has been the most prolific application of bilevel optimization models. Motto et al. (2005) implemented the duality-based reformulation for the analysis of electric grid security under disruptive threats. This bilevel problem was originally formulated by Salmeron et al. (2004) with the purpose of identifying the interdictions that maximize network disruptions. A bilevel formulation for the expansion of transmission networks was developed by Garces et al. (2009) to maximize the average social welfare over a set of lower-level problems representing different market clearing scenarios; they also implemented the duality-based reformulation. Ruiz et al. (2012) modeled electricity markets as an Equilibrium Problem with Equilibrium Constraints (EPEC) in which competing producers maximize their profit in the upper level and a market operator maximizes social welfare in the lower level; they use the duality-based reformulation to guarantee optimality of the lower level problem and obtain an equilibrium solution by jointly formulating the KKT conditions of all producers. A similar strategy that includes the combination of duality-based and KKT reformulations was used by Huppmann and Egerer (2014) to solve a three-level optimization problem that models the roles of independent system operators, regional planners, and supra-national coordination in the European energy system.

Another interesting application of bilevel optimization is the facility location problem in a duopolistic environment. The model presented by Fischer (2002) selects facilities among a set of candidate locations and considers selling prices as optimization variables, which leads to a nonlinear bilevel formulation. The problem is simplified to a linear discrete bilevel formulation under the assumption that Nash equilibrium is reached for the prices. The solution to the discrete bilevel optimization problem is obtained using a heuristic algorithm.

Bilevel optimization models have also found application in chemical engineering. Clark and Westerberg (1990) presented a nonlinear bilevel programming approach for the design of chemical processes and proposed algorithms to solve it. In their formulation, the upper level optimizes the process design and the lower level models thermodynamic equilibrium by minimizing Gibbs free energy. Burgard and Maranas (2003) used bilevel optimization to test the consistency of experimental data obtained from metabolic networks with hypothesized objective functions. In the upper level, the problem minimizes the square deviation of the fluxes predicted by the metabolic model with respect to experimental data, whereas the lower level quantifies the individual importance of the fluxes. A bilevel programming model for supply chain optimization under uncertainty was developed by Ryu et al. (2004); the conflicting interests of production and distribution operations in a supply chain are modeled using separate objective functions. They reformulate the bilevel problem in single-level after finding the solution of the lower-level problem as parametric functions of the upper-level variables and the uncertain parameters. Chu and You (2014) presented an integrated scheduling and dynamic optimization problem for batch processes. The scheduling problem, formulated in the upper level, is subject to the processing times and costs determined by the nonlinear dynamic lower-level problem. The bilevel formulation is transformed to a single level problem by replacing the lower-level with piece-wise linear response functions. They assert that the bilevel formulation can be used as a distributed optimization approach whose solutions can easily adapt to variation in the problem's parameters.

It should also be noted that bilevel programming for nonlinear models has been the subject of research in chemical engineering. Faisca et al. (2007) presented a multi-parametric programming approach that replaces the lower-level problem by its rational reaction set parametrized on the upper-level variables. For global optimization of continuous and mixed-integer bilevel problems, Kleniati and Adjiman (2015) developed the Branch-and-Sandwich algorithm, which solves bilevel programs with noncovex lower-level problems.

The novelty of our research resides on the application of bilevel optimization for capacity expansion planning in a competitive environment. Bilevel programming for these kind of problems can be seen as a risk mitigation strategy given the significant influence of external decision makers in the economic success of investment plans. In particular, we propose a mathematical model that includes a rational market behavior beyond the classic game theoretical models. The investment plans obtained from this approach are found to be less sensitive to changes in the business environment in comparison to the plans obtained from single-level models.

In order to solve the challenging bilevel formulation, we test the KKT and the duality-based reformulations with an illustrative example, a middle-size example, and an industrial example. The results show the advantages of the duality-based reformulation in terms of computational effort. Despite the efficiency obtained with this reformulation, we found necessary to implement two additional improvement strategies to solve large-scale instances.

The remaining of this article is organized as follows. In Section 2, we describe the problem. In Section 3, we present the single-level capacity planning formulation. Section 4 presents the bilevel capacity planning problem with rational markets. In Section 5, we