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A higher-order non-hydrostatic σ model for simulating non-linear refraction–diffraction of water waves

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ABSTRACT

A higher-order non-hydrostatic σ model is developed to simulate non-linear refraction–diffraction of water waves. To capture non-linear (or steep) waves, a 4th-order spatial discretization is utilized to approximate the large horizontal pressure gradient. A higher-order top-layer pressure treatment is further implemented to resolve wave propagation. The model's characteristics including linear wave dispersion and non-linearity are carefully examined. The accuracy of the present model using only two vertical layers is validated by laboratory data and the available results predicted by the non-linear Schrödinger equation, Boussinesq-type equations, the non-linear mild slope equation, and the Laplace equation. Features of harmonic generation as well as the influences of dispersion and non-linearity on wave energy transfer processes are discussed.

1. Introduction

Accurate modeling of surface wave propagation from deep water to shallow water is an important topic in coastal and ocean engineering (Mei and Liu, 1993; Janssen et al., 2006). As waves travel over irregular topographies, the wave form is modified owing to the effects of dispersion, shoaling, refraction, diffraction, and reflection. Particularly in the shallow-water regions, non-linear effects related to wave–bottom and wave–wave interactions can further complicate energy transfers and harmonic generation (Beji and Battjes, 1993; Kirby, 1997). Over the last several decades, a lot of attempts have been made to develop unified models capable of resolving combined refraction–diffraction of water waves with wide-ranging dispersion and non-linearity (Wu, 2001).

Three types of models, in general, are utilized to study water waves. First, the most widely-used are the so-called depth-integrated models, e.g. the standard Boussineq equation (Peregrine, 1967) for weakly non-linear shallow-water waves (Liu et al., 1985) or the classical mild-slope equation (Berkhoff, 1972) for linear waves (Kirby and Dalrymple, 1983). Further efforts have been made to extend the applicability of depth-integrated models, i.e. the modified Boussinesq-type models for deeper water (Madsen and Sørensen, 1992; Nwogu, 1993; Chen and Liu, 1995; Wei et al., 1995; Gobbi et al., 2000; Madsen et al., 2002; Lynett and Liu, 2004; Hsiao et al., 2005) or the non-linear mild slope models including the parabolic approximation (Kirby and Dalrymple, 1983; Liu and Tsay, 1984; Kaihatu and Kirby, 1995; Tang and Ouellet, 1997). One can refer to an excellent review by Liu and Losada (2002) for the development of depth-integrated models.

Second, the Laplace equation provides a framework to describe surface waves without limitations of water depth (Longuet-Higgins and Cokelet, 1976; Dommermuth and Yue, 1987; Dold, 1992; Guyenne and Grilli, 2006). For applications of this approach to steep water waves or non-linear refraction–diffraction problems, one can refer to thorough reviews by Tsai and Yue (1996). While the accuracy of this approach has been addressed, efficiently simulating 3D waves is still of concern (Li and Fleming, 1997; Fructus et al., 2005; Fochessato and Dias, 2006). In addition, the Laplace equation is based upon the potential flow theory, hindering the simulations of wave propagation with rotation and energy dissipation.

Third, the so-called non-hydrostatic model based upon the full Navier–Stokes equations is the most detailed approach for predicting non-linear dispersive waves. Normally this type of models (Mayer et al., 1998; Casulli, 1999; Li and Fleming, 2001; Namin et al., 2001; Lin and Li, 2002; Kocyigit et al., 2002; Chen, 2003) employs the hydrostatic pressure assumption at the top-layer. As a result, a relatively large number (10–40) of vertical layers are needed to resolve dispersive wave. In recent years, the feasibility of efficient non-hydrostatic models using a relatively small number of vertical layers have been proposed and demonstrated. By specifying the non-hydrostatic pressure at the free surface using the Kellor-box scheme (Stelling and Zijlema, 2003) or the integral method (Yuan and Wu, 2004a,b), these models with few vertical layers are capable of resolving wave dispersion to a certain degree. For instance, the two-layer model by Zijlema and Stelling (2008)

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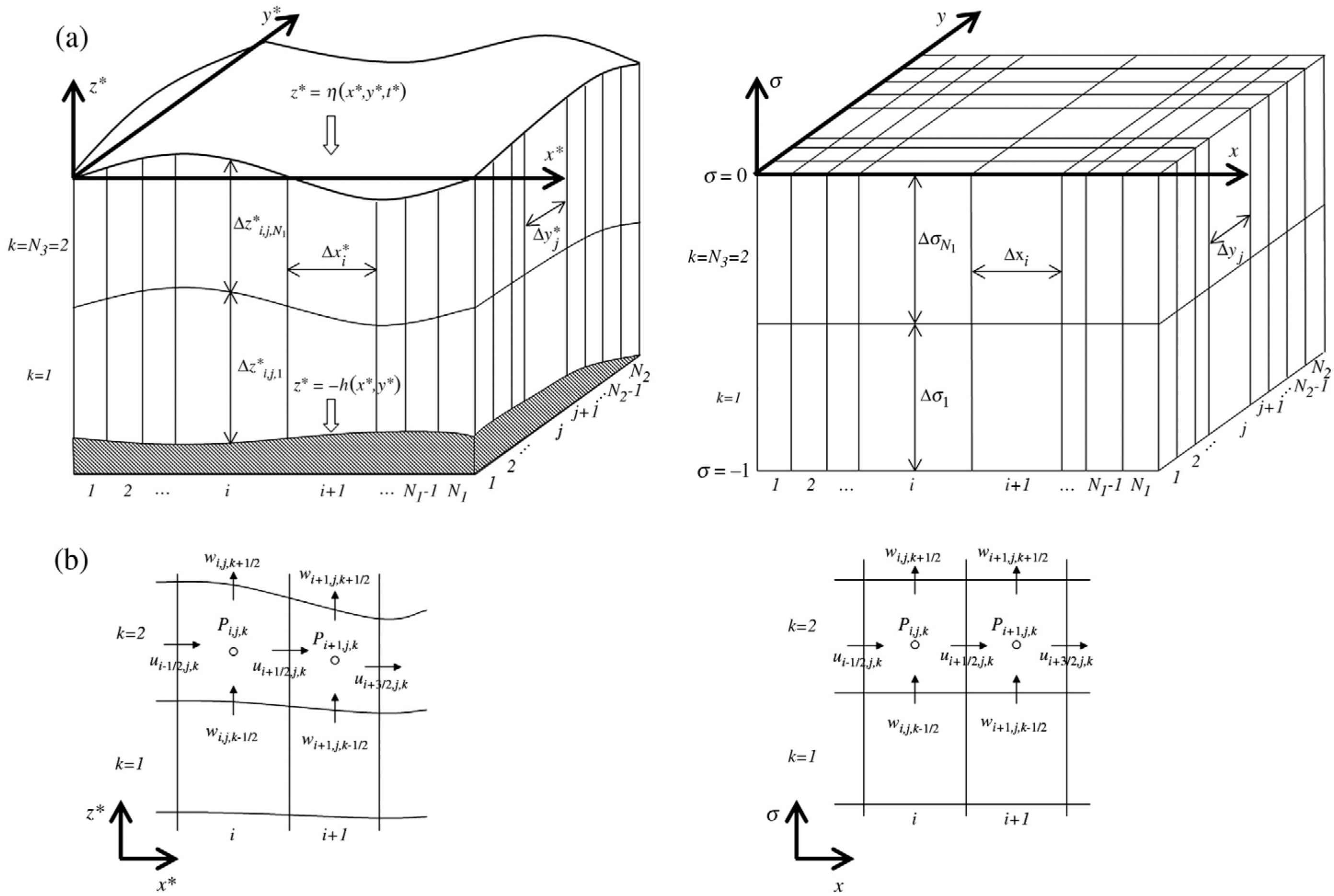

Fig. 1. (a) σ transformation and (b) staggered grid system between the Cartesian co-ordinate and the σ co-ordinate.

can reasonably resolve wave transformation over irregular topography but unfaithfully reveal the higher harmonics at Kh ~1.2 (see their Fig. 11), where K and h are the wave number and water depth. Yuan and Wu (2006) show that given a tolerance error of 1%, two-layer and five-layer models can capture wave dispersion of a dimensionless relative water depth $Kh = 1.0$ and $Kh = 5.0$, respectively. In other words the vertical layer number determines the accuracy of non-hydrostatic models in resolving wave dispersion. Interestingly it is found that the number of vertical layers does not strongly interplay with non-linearity once dispersion is resolved. There is an ongoing research on the development and application of efficient non-hydrostatic models in predicting nearshore wave transformation (Cea et al., in press; Young and Wu, 2009).

The objective of this paper is to develop an efficient higher-order, three-dimensional (3D) non-hydrostatic σ model for accurately resolving combined refraction–diffraction of water waves with wide-ranging dispersion and non-linearity. While the boundary-fitted σ co-ordinate system provides the advantage to well describe the free surface and bathymetry, relatively large numerical errors can result from the inappropriate treatment of the horizontal pressure gradient over steep surfaces (Haney, 1991). Following the approach in Young et al. (2007), a 4th-order approximation for the horizontal pressure gradient is utilized to resolve non-linear (or steep) waves. To better predict wave dispersion, a higher-order top-layer pressure treatment is further implemented. Model characteristics including dispersion and non-linearity are carefully examined. The higher-order 3D model is then applied to all the experimental cases in Whalin (1971). To the best of the authors' knowledge, application of non-hydrostatic models against these experiments has not yet been reported in the literature. We will compare the accuracy of the present model using only two vertical layers with the experimental data and results of several available models including the non-linear Schrödinger equation (Liu and Tsay, 1984), the Boussinesq-type equation (Liu et al., 1985; Rygg, 1988; Madsen and Sørensen, 1992), the non-linear mild slope equation (Kaihatu and Kirby, 1995; Tang and Ouellet, 1997), and the Laplace equation (Li and Fleming, 1997). Finally features of harmonic generation in each case as well as the influences of dispersion and non-linearity on wave energy transfer processes are discussed.

2. Non-hydrostatic model

2.1. Governing equations and boundary conditions

The governing equations for free-surface waves are the unsteady, incompressible, Navier–Stokes equations. To describe both the free-surface elevation and bottom topography, e.g. non-linear waves over irregular geometry, the time dependent Cartesian domain (x^*,y^*,z^*,t^*) is mapped into a stationary rectangular σ domain (x,y,σ,t) (Yuan and Wu, 2004a, Young et al., 2007), i.e.,

$$t = t^*, x = x^*, y = y^*, \sigma = \frac{z^* - \eta(x^*, y^*, t^*)}{h(x^*, y^*) + \eta(x^*, y^*, t^*)}, \tag{1}$$

yielding $\sigma = 0$ at the free surface $z^* = \eta(x^*,y^*,t^*)$ and $\sigma = -1$ at the bottom $z^* = -h(x^*,y^*)$, as seen in Fig. 1. The transformed governing equations in primitive variables are

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial \sigma}\frac{\partial \sigma}{\partial x^*} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial \sigma}\frac{\partial \sigma}{\partial y^*} + \frac{\partial w}{\partial \sigma}\frac{\partial \sigma}{\partial z^*} = 0, \tag{2}$$