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Short communication

Influence of discrete step size on wind setup component of storm surge

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ABSTRACT

Open coast storm surge water levels consist of wind setup due to wind shear at the water surface; a wave setup component caused by wind induced waves transferring momentum to the water column; an atmospheric pressure head component due to the atmospheric pressure deficit over the spatial extent of the storm system; a Coriolis forced setup or setdown component due to the effects of the rotation of the earth acting on the wind driven alongshore current at the coast; a possible seiche component due to resonance effects initiated by moving wind system, and, if astronomical tides are present, an astronomical tide component (although the tide is typically considered to be a forced astronomical event and not really a direct part of the external wind-driven meteorological component of storm surge). Typically the most important component of a storm surge is the wind setup component, especially on the U.S. East Coast and the Gulf of Mexico shorelines. In many approaches to storm surge modeling, a constant depth approximation is invoked over a limited step size in the computational domain. The use of a constant depth approximation has received little attention in the literature although can be very important to the resulting magnitude of the computed storm surge. The importance of discrete step size to the wind setup storm surge component is considered herein with a simple case computation of the wind setup component on a linear slope offshore profile. The present study findings show that the constant depth approximation to wind setup storm surge estimation is biased on the low side (except in extremely shallow water depths) and can provide large errors if discrete step size is not sufficiently resolved. Guidance has been provided on the error that one might encounter for various step sizes on different slopes.

1. Introduction

Physical components of open coast storm surge water levels (Harris, 1963) are as follows in typical (but not always primary) order of importance on most Atlantic and Gulf Coast continental shelf locations:

1.1) Wind shear where the forcing of the water towards the coast by onshore winds causes a build-up of water along the coast. This effect is generally referred to as “wind setup” although sometimes the term “wind setup” also includes Coriolis effects noted as a separate component below.
1.2) Wave setup is caused by wind induced waves transferring momentum to the water column which must be balanced by pressure and bottom friction forces (Dean and Walton, 2009). Wave setup can cause an increase in water level elevations on the order of 20% or more of the offshore breaking wave height and can thus be a significant portion of the overall storm water level rise.
1.3) Pressure deficit due to the low pressure storm system which creates an inverse barometer effect, the result of the lower pressures causing the water under the pressure deficit (from ambient pressure) to be elevated (Harris, 1963). A rule of thumb, based on hydrostatic considerations is 1 cm rise in ocean surface for every 1 mb drop in pressure below ambient pressure;
1.4) Coriolis force is a result of the rotation of the earth, and causes wind-driven currents in the Northern Hemisphere to be deflected to the right in a rotating frame of reference. Winds blowing parallel to the coast cause an increase in sea level along the coastline when the coast is to the right of the wind direction and a decrease in sea level when the coast is to the left of the wind direction.
1.5) Possible seiche component (Dean and Dalrymple, 1991; Kamphius, 2000) due to resonance effect of transients in moving wind system.

The first two components typically make up 75–90% of the storm surge on most mildly sloping offshore continental shelves along the East and Gulf Coasts of the U.S. although steep sloped bathymetry as experienced in many island situations and in many West Coast areas of the world would not adhere to this simplification.

This technical note will address the importance of the wind stress component of storm surge on the open coast and in particular the significance of the discrete step size used in characterizing offshore bathymetry in influencing the magnitude of the storm surge. In many approaches to storm surge modeling and numerical computations of

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storm surge, a constant depth approximation is invoked over a limited horizontal direction discrete step size in the computational domain. The use of a constant depth approximation for bathymetry has received limited attention in the literature although it can be very important to the resulting magnitude of the computed storm surge. The importance of discrete step size to the wind setup storm surge component is considered herein with special reference to computations of the wind setup component on a linear slope offshore profile where it is shown that the wind setup storm surge estimation is always biased on the low side compared to the analytic solution.

Additionally guidance has been provided on the error that one might encounter for various step sizes on various slopes.

2. Numerical model

Discussion of a simple bathystrophic storm surge finite difference numerical model is made herein to provide necessary background for discussion of numerical model discretization effects of such a model. A simple bathystrophic storm surge model was developed by Freeman, Baer and Jung (1957) and considers the onshore surge to be in balance with the onshore component of wind stress and the Coriolis force associated with the alongshore transport of water. The alongshore transport of water, Q_y, is considered to vary with time. The governing equations are:

$$\frac{\partial\overline{\eta}}{\partial x} = \frac{1}{\rho g(h+\overline{\eta})}\left(n\tau_{wx} - f_c Q_y\right) \tag{1}$$

and

$$\frac{\partial Q_y}{\partial t} = \frac{\tau_{wy}}{\rho} - \frac{f|Q_y|Q_y}{8(h+\overline{\eta})^2} \tag{2}$$

in which $\overline{\eta}$ is the surge above a reference datum, τ_{wx} and τ_{wy} are the wind stress components in the x and y directions, respectively, ρ is the mass density of water, h is the local water depth, g is the gravitational acceleration, n is a factor (taken here as 1.15) which accounts approximately for the bottom shear stress on the water column in the same direction as that on the water surface, f_c is the Coriolis parameter ($=2\Omega\sin(\phi)$) where Ω is the Earth's rotation rate ($=7.27\times10^{-5}$ rad/s), ϕ is latitude, f is the non-dimensional bottom friction coefficient ($=0.08$) and Q_y is the alongshore transport of water in the y direction as before. The directions x and y form a right-handed coordinate system with x directed onshore.

The wind stress components, τ_{wx} and τ_{wy} are given by $\tau_{wx} = \rho KW^2\cos\theta$ and $\tau_{wy} = \rho KW^2\sin\theta$ where W and θ are the wind speed and direction, respectively.

The wind stress coefficient, is defined by the Van Dorn (1953) relationship

$$K = 1.2\times10^{-6},\ |W|\le W_c \tag{3a}$$

or

$$K = 1.2\times10^{-6} + 2.25\times10^{-6}\left(1-\frac{W_c}{|W\}}\right),\ |W|\ge W_c \tag{3b}$$

and W_c is a critical wind speed (=5.6 m/s [=18.4 ft/s]). The Van Dorn (1953) expression was utilized herein for wind shear stress as the purpose of the present paper is to note importance of discrete step size in a numerical model rather than focus on wind shear stress relationships.

Eqs. (1) and (2) are typically cast into finite difference form for solution in a numerical program utilizing bathymetry that is stepped across the offshore profile, with a constant depth approximation consisting of the average depth within a (Δx, Δy) grid square utilized for the step size. One such numerical model utilizing constant depth over discrete step (see Walton and Dean, in press) has been benchmarked against analytic results and utilized to assess the importance of shelf bathymetry on storm surge.

3. Analytical model

3.1. Linear slope solution

Dean and Dalrymple (1991) have provided analytic solutions for the case of the wind shear stress component of storm surge acting on both a constant depth shelf and a linear slope shelf which will be further investigated herein and utilized for comparison with numerical (constant depth over grid) approximate results.

The non-dimensional solution form for the wind setup component ($=\overline{\eta}$) on the linear slope shelf is provided as follows:

$$\frac{x}{l} = \left(1-\frac{h+\overline{\eta}}{h_0}\right) - A\cdot\ln\left(\frac{(h+\overline{\eta})/h_0 - A}{1-A}\right) \tag{4}$$

where x = distance from offshore boundary at $x=0.0$; l = width of shelf; h = depth of water; h_0 = depth of water at offshore boundary (seaward edge of shelf); $\overline{\eta}$ = wind stress setup; and $A = n\tau_{wx}l/\rho g h_0^2$; τ_{wx} = the wind shear stress as defined in Dean and Dalrymple (1991); ρ = density of seawater; and g = the acceleration of gravity. It should be noted that the solution is implicit in $h+\overline{\eta}$ and cannot be solved directly. A solution graph is provided in Dean and Dalrymple (1991) for the wind shear stress setup on the shelf region $x=l$ for two cases of $A=0.01$ and $A=0.05$. An additional requirement of the analytic solution provided in Eq. (4) is that either $A<1$ or that (in somewhat pathological circumstances from a practical engineering viewpoint) $A\ge1$ and $\frac{h+\overline{\eta}}{h_0}\ge A$ in which case $h+\overline{\eta}$ would be constant (for $A=1$) or would have a slope greater than the bottom slope (for $A>1$). In the present paper the solution is desired for the intersection of the still water level (SWL) with the shoreline (i.e. at $x=l$).

3.2. Constant depth solution

A similar but more straightforward solution over the entire shelf region is provided in Dean and Dalrymple (1991) for the constant depth case in dimensionless form as follows:

$$\frac{\overline{\eta}}{h_c} = \sqrt{1+2A_c\frac{x}{l}} - 1 \tag{5}$$

where h_c the constant shelf depth over the domain and A_c = corresponding non-dimensional shear stress as defined previously for linear slope case (only with constant depth h_c rather than h_o). In the situation where the integration is over a discrete step, the constant depth solution can be generalized to the dimensional form as follows:

$$\frac{\Delta\overline{\eta}}{h_c} = \sqrt{1+2A_c\frac{\Delta x}{l}} - 1 \tag{6}$$

where $\Delta\overline{\eta}$ refers to the setup difference over the step of size Δx. Utilizing this "exact" analytic solution for the constant depth case over a discrete step, one can numerically calculate the "exact" storm surge (within computer computational error) over a discretized approximation to a linear slope using discrete step size Δx and stepwise constant depth set to the average depth over the step size. This "exact" computation of an approximate representation of the slope will provide an approximate answer to the "exact" answer of the linear slope storm surge setup.

4. Analytic error analysis

Through examination of the wind setup over a single grid, it will be shown that the wind setup error in approximating the grid depth as