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Numerical solutions of the sediment conservation law; a review and improved formulation for coastal morphological modelling

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Abstract

Numerical solutions of the sediment conservation law are reviewed in terms of their application to bed update schemes in coastal morphological models. It is demonstrated that inadequately formulated numerical techniques lead to the introduction of diffusion, dispersion and the bed elevation oscillations previously reported in the literature. Four different bed update schemes are then reviewed and tested against benchmark analytical solutions. These include a first order upwind scheme, two Lax–Wendroff schemes and a non-oscillating centred scheme (NOCS) recently applied to morphological modelling by Saint-Cast [Saint-Cast, F., 2002. Modelisation de la morphodynamique des corps sableux en milieu littoral (Modelling of coastal sand banks morphodynamics), University Bordeaux I, Bordeaux, 245 pp.]. It is shown that NOCS limits and controls numerical errors while including all the sediment flux gradients that control morphological change. Further, no post solution filtering is required, which avoids difficulties with selecting filter strength. Finally, NOCS is compared to a recent Lax–Wendroff scheme with post-solution filtering for a longer term simulation of the morphological evolution around a trained river entrance.

Keywords: Conservation laws; Spatial oscillations; Feedback; Coastal morphological modelling; Lax–Wendroff schemes; Non-oscillating central schemes

1. Introduction

Modelling coastal morphological evolution presents significant challenges in terms of both the accurate representation of physical processes and the ability to propagate changes in bathymetry over broad space and time scales. Typically, morphological models utilise process based sub-models for waves, currents and sediment transport, coupled with the sediment conservation law to determine changes in bed elevation (Roelvink and Stive, 1989; de Vriend et al., 1993; Nairn and Southgate, 1993; Plant et al., 2001; Johnson and Zyserman, 2002; van Rijn et al., 2003). While errors in the forcing sub-models will always exist, proper coupling of the morphology and hydro-sediment dynamics is essential for longer term modelling, particularly if the influence of changing environmental parameters or coastal development are to be assessed accurately. Accurate representation of the feedback between the bathymetry and the forcing is therefore required (Yu and Mei, 2000; Plant et al., 2004). However, the computational techniques required for broad-scale bathymetric modelling may lead to inaccuracies and instabilities in the predicted bed level, and these typically grow during model simulations (Johnson and Zyserman, 2002). These instabilities can arise irrespective of how well the physical process are characterised.

Changes in bathymetry are governed by the sediment conservation law, which for 1D sediment transport is

$$\frac{\partial Z}{\partial t}+\frac{\partial F}{\partial x}=0 \tag{1}$$

where Z is the level of the sea bed, x is the horizontal coordinate and F is the sediment flux for a given porosity. Despite the simplicity of Eq. (1) and because of the mathematical nature of F, equations of this form are difficult to solve numerically, with

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some apparently simple and memory efficient schemes being unconditionally unstable (Press et al., 1992). In particular, the numerical scheme needs to avoid uncontrolled dispersion and diffusion. The former leads to the generation of spurious numerical oscillations, which drastically restricts the validity of the computed solution from long-term to very short-term. The latter leads to artificially reduced sand wave amplitudes, which, combined with non-linear sediment transport, reduces sand wave migration speed. Hence, given the sensitivity of long term morphological predictions to initial conditions (Blondeaux, 2001), it is very important that any numerical errors are minimised. For example, Johnson and Zyserman (2002) argued that the resulting high wave number spatial oscillations would reduce morphological predictions to noise if left unchecked.

Bed level instabilities in numerical calculations can occur as a result of either poor representation of physical processes, or from ill behaved numerical schemes. The pioneering work of Dally and Dean (1984) illustrated an example of a bed level instability generated from their turbulence formulations which lead to an overestimation of suspended sediment flux. When coupled with the converging flow around the breaker line the result was a very narrow tall bar crest in comparison to laboratory measurements. Instabilities of this form arise primarily as a result of missing interaction between different hydrodynamic and sediment transport processes (Nairn and Southgate, 1993). Numerical instabilities may suppress or enhance process driven instabilities, and this is complicated by the highly variable temporal and spatial scales of the instabilities. With complex bathymetry, it may then be difficult to judge when numerical errors lead to bed level oscillations that dominate over real variations in the bed topography. An example of this is illustrated later in Section 6. However, we show here that growth of such instabilities only occurs when they are propagated by unstable or dispersive numerical schemes, and, typically, such schemes have been applied to solve (1). In addition, it is more rigorous to choose an appropriate numerical scheme which avoids introducing instabilities in the first instance.

The present paper addresses this issue and demonstrates that inadequately formulated techniques for solving Eq. (1) lead to numerical errors resembling diffusion and dispersion. Four different bed update schemes are then reviewed and tested against benchmark analytical solutions. These are; a simple first order scheme, FOS, (Courant et al., 1952); a Lax–Wendroff scheme with slope limiters, LW TVD, (Vincent and Caltagirone, 1999); a Lax–Wendroff scheme with and without post solution filtering, JZ2002, (Johnson and Zyserman, 2002); and a non-oscillating centred scheme, NOCS, (Saint-Cast, 2002) based on Jiang and Tadmor (1998). The review is limited to one-dimensional (1D) formulations for simplicity, with the two-dimensional (2D) versions of these schemes given in the original references above. However, the numerical testing includes both 1D and 2D simulations and shows no significant differences between 1D and 2D scheme performances. We show that NOCS limits and controls numerical errors while including all the sediment flux gradients that control morphological change. Further, no post solution filtering is required. The inclusion of bed slope effects as suggested by de Vriend (1986) and Watanabe (1988) is also no long required to additionally control numerical instability. Subsequently, the two best performing schemes (JZ2002 and NOCS) are compared for a more complex problem.

The paper is arranged as follows. Section 2 below provides some further background relevant to this work. In Section 3 we derive the equivalent homogenous equations for the exact, first and second order numerical approximations of Eq. (1) for the case of sediment flux linearly varying with bed level. The sources and behaviour of numerical instabilities are also clarified. Section 4 presents an overview of the performance and limitations of the four bed update schemes. The results of the benchmark testing are presented in Section 5, and these highlight errors due to diffusion, dispersion, grid orientation bias and the generation of high wavenumber oscillations. Section 6 compares JZ2002 and NOCS during a longer term simulation of the morphological evolution around a trained river entrance, a more physically realistic problem. Final conclusions are given in Section 7.

2. Background

2.1. Types of morphological models

A wide range of different models have been used to predict coastal morphology. These include conceptual models, process-based one-dimensional models for shoreline, contour or profile evolution, through to two-dimensional horizontal (2D) and quasi three-dimensional (Q3D) process-based models for complex bathymetry, e.g., tidal entrances. Examples of conceptual models are; for inlets Balouin et al. (2001), Gravens (1997), Kana et al. (1999), Kraus (2000), and Michel and Howa (1997); for rip currents Brander (1999b,a); and for nearshore bars Ruessink and Terwindt (2000) and Aagaard (2002). Examples of shoreline, one-line and profile models are Hanson and Larson (1987), Brøker-Hedegaard et al. (1991), Nairn and Southgate (1993), Steetzel et al. (2000), and Li et al. (2002). Examples of 2D and Q3D process-based models are DHI's MIKE21 CAMS (Johnson, 1994), Delft3D (Roelvink and Banning, 1994), Ranasinghe et al. (1999) and MORPHODYN (Saint-Cast, 2002).

These three basic groups of models differ in the aspects of the coastal morphology assessed and the numerical effort utilised. Profile models and 2D/Q3D models differ as a result of different longshore and cross-shore length scales, with the possible longshore propagation paths limited to the length of continuous sandy coast line, while cross shore propagation paths are limited to between the run-up limit and the depth of closure. The computational effort required by conceptual, profile/shoreline and 2D/Q3D modelling approaches is negligible, moderate and extreme, respectively. Consequently, implementations of 2D/Q3D models minimise the computational effort by increasing spatial and temporal discretisations, leading to lower resolution of sand wave features when compared to profile models. This reduction in resolution is counter balanced by the inclusion of second order accurate