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Estimation of boundary-layer flow of a nanofluid past a stretching sheet: A revised model*

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Abstract: The previous model for the boundary layer nanofluid flow past a stretching surface with a specified nanoparticle volume fraction on the surface is revisited. The major limitation of the previous model is the active control of the nanoparticle volume fraction on the surface. In a revised model proposed in this paper, the nanoparticle volume fraction on the surface is passively controlled, which accounts for the effects of both the Brownian motion and the thermophoresis under the boundary condition, whereas the Buongiorno's model considers both effects in the governing equations. The assumption of zero nanoparticle flux on the surface makes the model physically more realistic. In the revised model, the dimensionless heat transfer rates are found to be higher whereas the dimensionless mass transfer rates are identically zero due to the passive boundary condition. It is also found that the Brownian motion parameter has a negligible effect on the Nusselt number.

Key words: boundary layer flow, nanofluid, stretching sheet, Brownian motion, thermophoresis

Introduction

Buongiorno[1] developed a model for nanofluid that includes both the Brownian motion and thermophoresis effects. This model was employed by Kuznetsov and Nield[2] and Nield and Kuznetsov[3] to examine the influence of nanoparticles on the free convection past a vertical plate. They employed boundary conditions with respect to the nanoparticle fraction akin to the temperature. Later on, Khan and Pop[4] employed the same model to investigate the laminar flow generated by the stretching of a flat surface. They studied the effects of Brownian and thermophoresis parameters on the dimensionless heat and mass transfer rates using the same approach as used in Refs.[2,3]. This means that the nanoparticle fraction on the wall can be specified arbitrarily, which is not realistic physically. Most recently, Kuznetsov and Nield[5,6] developed a physically realistic type of boundary condition which accounts for the effect of both Brownian and thermophoresis parameters. According to this new type of boundary condition, there is zero nanoparticle flux on the surface and the particle fraction values are adjusted accordingly. The model developed in Kuznetsov and Nield[5,6] was employed by Khan et al.[7,8].

Pal and Mandal[9] studied the magnetohydrodynamic boundary layer flow of an electrically conducting convective nanofluids induced by a non-linear vertical stretching/shrinking sheet with viscous dissipation, thermal radiation, and Ohmic heating. Their results reveal that by increasing the value of the Hartman number the velocity will decrease, whereas a reverse effect is found in the temperature profiles. Das[10] investigated numerically the boundary layer

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flow of a nanofluid over non-linear permeable stretching sheet with prescribed surface temperature in the presence of partial slip. Sandeep et al.[11] analyzed the flow, the heat and the mass transfer behavior of Jeffrey, Maxwell and Oldroyd-B nanofluids over a permeable stretching sheet in the presence of transverse magneticfield, thermophoresis, Brownian motion and suction/injection. They found that the friction factor of the Maxwell nanofluid is less important as compared with the Oldroyd-B and Jeffrey nanofluid. Chandrasekar and Kasiviswanathan[12] applied a variational technique to the MHD, the radiative nanofluid flow over a non-isothermal stretching sheet with Brownian motion and thermophoresis effects by Gyarmati's principle. The heat and mass transfer effects were investigated and analyzed by this technique. Ganga et al.[13] analyzed the effects of the space and temperature dependent internal heat generation/absorption on the magnetohydrodynamic boundary layer flow of the water based nanofluid over a stretching sheet with different nanoparticles. The influences of the nanoparticle volume fraction, the magnetic field, the Prandtl number, the non-uniform heat source/sink, the local skin friction coefficient and the reduced Nusselt number were investigated for different nanoparticles. Khan et al.[14] investigated the problem of the oblique hydromagnetic stagnation point flow of the electrically conducting optically dense viscous incompressible nanofluid of a variable viscosity over a convectively heated stretching sheet in the presence of thermal radiation. They analyzed the effects of various controlling parameters on the dimensionless velocity, temperature, nanoparticles concentration, skin friction, Nusselt and Sherwood numbers. Makinde et al.[15] investigated the combined effects of the thermal radiation, the thermophoresis, the Brownian motion, the magnetic field and the variable viscosity on the boundary layer flow, the heat and the mass transfer of an electrically conducting nanofluid over a radially stretching convectively heated surface. Their results reveal that the heat transfer rate is reduced with the increase of the viscosity and the nanofluid parameters whereas the mass transfer rates are enhanced with the increase of the Brownian motion parameter and the Lewis number. Ahmad[16] studied a classical non-Newtonian fluid in the presence of nano-particles over a non-linear stretching sheet. The correlation expressions for the skin friction, the Nusselt number and the Sherwood number were developed by performing a linear regression on the obtained numerical data. Mabood and Khan[17] obtained an analytical solution of an unsteady two-dimensional MHD nanofluid flow with heat and mass transfer over a heated surface. They conducted a detailed study illustrating the influences of the magnetic, unsteady, suction/injection and nanofluid parameters, on the dimensionless velocity, temperature, concentration as well as on the skin friction coefficient, and the reduced Nusselt and Sherwood numbers.

In this study, the model of the nanofluid flow past a stretching sheet in Ref.[4] is revised with the assumption that the value of nanoparticles at the wall are no longer given.

1. Formatting mathematical model

The present analysis closely follows the work of Ref.[4] and so only a brief outline is given here. A two-dimensional steady boundary layer flow of a nanofluid past a stretching surface is under consideration. The sheet is linearly stretched with linear velocity $u_w(x) = ax$, where a is a constant and a steady uniform stress is accompanied by a pair of forces of equal magnitude and opposite directions along the x-axis so that the sheet is stretched with the origin kept fixed. The flow takes place at $y \geq 0$, where y is the coordinate measured normal to the stretching surface. It is assumed that on the stretching surface, the temperature T keeps a constant value T_w and the nanoparticle fraction C takes the value C_w for the actively controlled nanoparticles on the surface whereas C is passively controlled by the effects of thermophoresis for the passively controlled nanoparticles. The ambient values, attained as y tends to infinity, of T and C are denoted by T_∞ and C_∞, respectively.

The basic steady conservation equations of mass, momentum, thermal energy and nanoparticles for the nanofluids can be written in Cartesian coordinates x and y as (see Kuznetsov and Nield[5,6])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$\begin{aligned} u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \\ \tau\left\{D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x} + \frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) + \right. \\ \left.\left(\frac{D_T}{T_\infty}\right)\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right]\right\} \end{aligned} \tag{4}$$