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Numerical solution of thermo-solutal mixed convective slip flow from a radiative plate with convective boundary condition*

M. J. UDDIN[1*], O. Anwar BÉG[2], M. N. UDDIN[3], A. I. Md. ISMAIL[4]
1. Department of Mathematics, American International University-Bangladesh, Dhaka, Bangladesh,
E-mail: jashim_74@yahoo.com
2. Fluid Mechanics, Spray Research Group, Petroleum and Gas Engineering Division, Room G77, Newton Building, School of Computing, Science and Engineering (CSE), University of Salford, M54WT, UK
3. Department of Mathematical Sciences, Ball State University, 2000 W University Avenue, Muncie, IN 47306, USA
4. School of Mathematical Sciences, Universiti Sains Malaysia, Penang 11800, Malaysia

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Abstract: A mathematical model for mixed convective slip flow with heat and mass transfer in the presence of thermal radiation is presented. A convective boundary condition is included and slip is simulated via the hydrodynamic slip parameter. Heat generation and absorption effects are also incorporated. The Rosseland diffusion flux model is employed. The governing partial differential conservation equations are reduced to a system of coupled, ordinary differential equations via Lie group theory method. The resulting coupled equations are solved using shooting method. The influences of the emerging parameters on dimensionless velocity, temperature and concentration distributions are investigated. Increasing radiative-conductive parameter accelerates the boundary layer flow and increases temperature whereas it depresses concentration. An elevation in convection-conduction parameter also accelerates the flow and temperatures whereas it reduces concentrations. Velocity near the wall is considerably boosted with increasing momentum slip parameter although both temperature and concentration boundary layer thicknesses are decreased. The presence of a heat source is found to increase momentum and thermal boundary layer thicknesses but reduces concentration boundary layer thickness. Excellent correlation of the numerical solutions with previous non-slip studies is demonstrated. The current study has applications in bio-reactor diffusion flows and high-temperature chemical materials processing systems.

Key words: slip flow, thermal radiation, lie group theory, heat source/sink, materials processing

Introduction

Slip flows occurs in a diverse range of technological applications including sheet processing[1], nuclear power systems[2], and foam production[3] and fluidic cells in medicine[4]. Slip flows require a modification of the classical "no-slip" velocity boundary condition at a wall with a "slip boundary condition" and have stimulated considerable interest in recent years. Ulmanella and Ho[5] found via experiments, the velocity slip for various micro-sized channels. The velocity at the wall is a function of shear rate, type of liquid and surface morphology. The possibility of temperature jump along with velocity slip was described by Bocquet and Barrat[6]. The velocity slip and temperature jump was modelled by Bocquet and Barrat[6] via the introduction of velocity slip length and temperature slip length terms, respectively. Many analytical and numerical methods have been employed to simulate a wide spectrum of slip flows. Wang[7] reported on the interaction of surface (momentum) slip on rotating Poiseuille and Couette flows, showing that for the former case slip enhances longitudinal flow rate at low rotation, but decreases it at high rotation, whereas in the latter case, longitudinal drag is reduced with greater slip. Nandy[8] studied unsteady flow of Maxwell fluid toward a permeable shrinking surface with Navier slip. Effects of hydrodynamic slip on the steady flow of an incompressible electrically conducting fluid

* **Biography:** M. J. UDDIN (1974-), Male, Ph. D., Associate Professor

past a channel with porous plates are reported by Ibáñez[9].

Double-diffusive (thermo-solutal) convection flows are also of interest in many branches of engineering sciences including energy storage[10], nuclear reactor leakage hazards[11], nano-technological materials processing[12] and heat exchangers[13]. In natural convection heat and mass transfer, thermal and species buoyancy effects play a significant role. Slip flows with combined heat and mass transfer have also received some attention, primarily in chemical engineering applications. Wu and Chen[14] discussed transverse uniformity of concentration distribution of a solute in a solvent flowing along a straight pipe. They used Mei's homogenization technique to find solutions for the concentration transport. Wu et al.[15] presented concentration distribution of contaminant transport in wetland flows. They presented an analytical solution for the multi-dimensional concentration distribution by the method of mean concentration expansion.

Thermal radiation heat transfer arises in many chemical engineering processes. At a high temperature, thermal radiation may change the distribution of temperature in the boundary layer and this affects the heat transfer at the wall. A variety of radiative heat transfer models have been utilized for transport modelling. Due to the complexity of solving the integral-differential equation of radiative transfer these models are frequently algebraic flux approximations. Radiative flux was observed to significantly modify the critical Rayleigh and wave numbers and affect convection strongly. Mahmoud[16] employed the Rosseland diffusion flux model to study mixed convection-radiation flow from a horizontal permeable surface aligned parallel to a uniform free stream. Gupta et al.[17] used a variational finite element method and Rosseland flux model to simulate non-Newtonian radiative-convection flow from a shrinking polymeric sheet showing that the flow is accelerated substantially with increasing thermal radiation effect which also supplements energy transfer to the flow and enhances thermal boundary layer thickness. The Rosseland diffusion model was also implemented by Khan et al.[18] for radiative nanofluid slip flow.

In the present study a mathematical model is developed for buoyancy-driven mixed thermal convection-radiation slip flow from a vertical plate with species diffusion. A convective surface boundary condition is also prescribed and heat source/sink effects included. The Rosseland diffusion flux model is used for radiative heat transfer. Via Lie group theory methods the conservation equations are rendered into similarity ordinary differential equations and numerical solutions developed with shooting method. Selected solutions are verified with comparison to earlier non-slip flows.

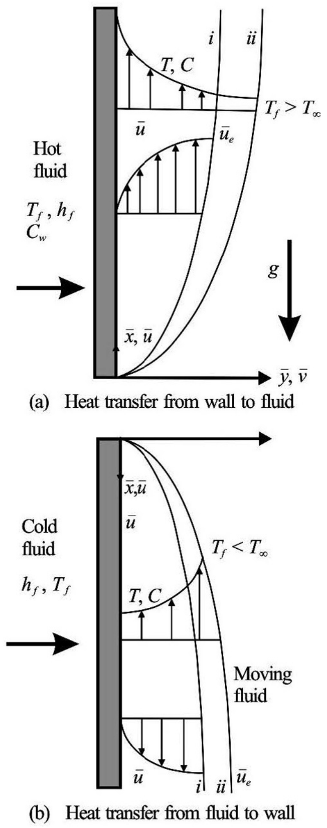

Fig.1 Flow configuration and coordinate system

1. Mathematical flow model

Newtonian viscous incompressible double-diffusive mixed convection is studied. The geometry and the rectangular coordinates, $\bar{x}$ and $\bar{y}$, and the corresponding velocity components, $\bar{u}$ and $\bar{v}$ and flow configuration are illustrated in Fig.1 (in which i represents momentum while ii represents thermal and concentration boundary layers, in general thermal and concentration boundary layer thickness are not the same). It is assumed that the uniform temperature of the ambient fluid is T_∞, the unknown temperature of the plate is T_w and the left surface of the plate is heated from a hot fluid of temperature $T_f\ (> T_\infty)$ or is cooled from a cooled fluid $(T_f < T_\infty)$ by the process of convection. This then yields a heat transfer variable coefficient $h_f(\bar{x}/L)$. It is assumed that the thermal radiation is present in the form of a uni-directional flux, applied transverse to the wall surface and obeys the Rosseland diffusion approximation. This formulation allows the transformation of the governing integral-differential equation for radiative energy balance into a Fourier-type diffusion equation analogous to that describing heat conduction or electrostatic pote-