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A robust WENO scheme for nonlinear waves in a moving reference frame*

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Abstract: For robust nonlinear wave simulation in a moving reference frame, we recast the free surface problem in Hamilton-Jacobi form and propose a Weighted Essentially Non-Oscillatory (WENO) scheme to automatically handle the upwinding of the convective term. A new automatic procedure for deriving the linear WENO weights based on a Taylor series expansion is introduced. A simplified smoothness indicator is proposed and is shown to perform well. The scheme is combined with high-order explicit Runge-Kutta time integration and a dissipative Lax-Friedrichs-type flux to solve for nonlinear wave propagation in a moving frame of reference. The WENO scheme is robust and less dissipative than the equivalent order upwind-biased finite difference scheme for all ratios of frame of reference to wave propagation speed tested. This provides the basis for solving general nonlinear wave-structure interaction problems at forward speed.

Key words: nonlinear waves, Weighted Essentially Non-Oscillatory (WENO), finite difference

Introduction

Exploiting the advantages of the high-order finite difference method in solving nonlinear potential flow, wave-structure interaction problems for ships travelling with steady forward speed, requires robust numerical methods. The proposed method extends previous works by Engsig-Karup et al.[1,2], Lindberg et al.[3] and Afshar et al.[4] on developing robust strategies for efficient modeling of nonlinear waves and linear and nonlinear wave-structure interaction. When the linearized problem is considered, a one-point upwind-biased approximation of the convective derivatives in the free surface boundary conditions is sufficient to ensure stability[5]. For nonlinear problems however, this is not found to be robust for all combinations of ship speed and wave celerity. In this paper, a simplified version of the Weighted Essentially Non-Oscillatory (WENO)[6] finite difference scheme is proposed for stabilizing the solution of the fully nonlinear problem. This scheme is tested using the nonlinear wave propagation problem in a moving frame of reference which mimics a forward speed seakeeping problem. The results of these strenuous numerical experiments indicate that it is robust and accurate.

1. A simplified WENO finite difference scheme for the convective terms

The scheme is presented in one dimension, but for 3-D problems it is applied in the same way to both of the horizontal convective terms. The WENO scheme creates two approximations of the convective derivative $\phi_{x,i} = (\partial \phi / \partial x)\big|_{x=x_i}$ using a left and a right biased stencil as shown in Fig.1.

These two approximations are combined with an appropriate flux to give the final result. Each of these stencils consists of r sub-stencils. On a left biased stencil, WENO 3 $(r=3)$ computes three third order approximations $\phi_{x,i}^{-s}$ to $\phi_{x,i}$ based on each of the $s = 0, \cdots, r-1$ sub-stencils. These approximations are combined using non-linear weights ω_s to compute the final result

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$$\phi_{x,i}^{-} = \sum_{s=0}^{r-1} \omega_s \phi_{x,i}^{-s} \tag{1}$$

The ω should ensure that when the solution is locally smooth, an order $(2r-1)$ accurate approximation is obtained. If the solution is discontinuous within the range of support of one or more of the stencils, the contributions from those stencils are weighted to zero and the accuracy of the final scheme reduces to r.

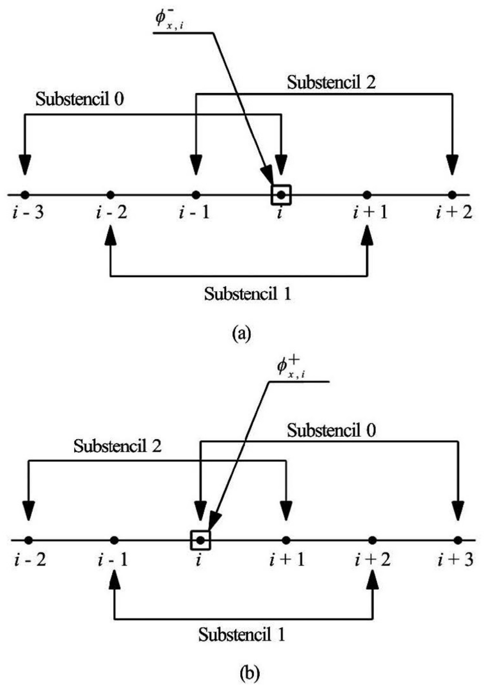

Fig.1 WENO 3 left and right biased stencils

Following Yamaleev and Carpenter[7] the nonlinear weights are defined as:

$$\omega_s = \frac{a_s}{\sum_{s=0}^{r-1} a_s}, \quad s = 0, \cdots, r-1, \quad a_s = d_s\left(1 + \frac{\tau}{\varepsilon + \beta_s}\right) \tag{2}$$

The d_s here are constant linear weights which, in the case of a smooth solution, will provide the order ($2r-1$) accurate result. The weights must sum to one, $\sum_{s=0}^{r-1} d_s = 1$.

Tabulated values of d_s for several specific cases can be found in the literature by Jiang and Shu[8], Balsara and Shu[9]. We present here a simple and general derivation procedure, applicable to any order of accuracy and easily implemented in a computer program. We seek r coefficients which sum to one and set to zero the first $r-1$ truncation error terms in the Taylor series expansion of the combined derivative approximation. Thus for $r = 3$, we have the system of equations

$$\begin{bmatrix} S_{0,1} & S_{1,1} & S_{2,1} \\ S_{0,2} & S_{1,2} & S_{2,2} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} d_0 \\ d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

where $S_{s,j}$, $s = 0, \cdots, r-1$ and $j = 1, \cdots, r-1$ represents truncation error coefficient j on sub-stencil s. For example, the derivative approximation on left sub-stencil 0 of order $r = 3$ is given by

$$\frac{1}{6\Delta x}(-2\phi_{i-3} + 9\phi_{i-2} - 18\phi_{i-1} + 11\phi_i) =$$

$$\phi_i^{(1)} - \frac{1}{4}\Delta x^3 \phi_i^{(4)} + \frac{1}{3}\Delta x^4 \phi_i^{(5)} + \cdots$$

where $\phi_i^{(n)}$ indicates the exact nth derivative of ϕ at grid point i Thus, $S_{0,1} = -1/4$ and $S_{0,2} = 1/3$. The set up and solution of this system can be easily implemented in a short computer program to provide the d_s for any choice of r.

The β_s in Eq.(2) are "smoothness indicators" which measure the smoothness of the solution on each stencil. Here again, sets of tabulated weights for specific cases can be found in Jiang and Shu[8], Balsara and Shu[9] based on rather complicated derivation procedures. We propose here a simple smoothness indicator that can be easily calculated numerically at arbitrary order

$$\beta_s = \sum_{l=2}^{r} [\phi_{s,i}^{(l)} \Delta x^{l-1}]^2 \tag{3}$$

This is a sum of all possible higher derivatives on the stencil, scaled to have units of velocity (m/s) and is thus a measure of the smoothness of the velocity. It becomes large whenever a discontinuity exists in the solution and thus forces the associated weight to become small for that stencil. The factor τ in Eq.(2) is defined as the squared normalized highest order derivative on the full stencil of $2r$ points,

$$\tau = \sum_{i=1}^{2r} [\phi_i^{(2r-1)} \Delta x^{2r-1}]^2 \tag{4}$$

The term ε in Eq.(2) is included to avoid dividing by zero and is set to 10^{-6}.

This expression for the nonlinear WENO weights (Eq.(2)) is chosen instead of the classical weights by Liu et al.[6] because it is found to introduce less numerical dissipation into the nonlinear wave solution. For