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Numerical simulation of self-similar thermal convection from a spinning cone in anisotropic porous medium*

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Abstract: Self-similar steady natural convection thermal boundary layer flow from a rotating vertical cone to anisotropic Darcian porous medium is investigated theoretically and numerically. The transformed non-dimensional two-point boundary value problem is reduced to a system of coupled, highly nonlinear ordinary differential equations, which are solved subject to robust surface and free stream boundary conditions with the MAPLE 17 numerical quadrature software. Validation with earlier non-rotating studies is included, and also further verification of rotating solutions is achieved with a variational finite element method (FEM). The rotational (spin) parameter emerges as an inverse function of the Grashof number. The influence of this parameter, primary Darcy number, secondary Darcy number and Prandtl number on tangential velocity and swirl velocity, temperature and heat transfer rate are studied in detail. It is found that the dimensionless tangential velocity increases whilst the dimensionless swirl velocity and temperature decrease with the swirl Darcy number, tangential Darcy number and the rotational parameters. The model finds applications in chemical engineering filtration processing, liquid coating and spinning cone distillation columns.

Key words: self-similarity, spinning cone, finite element method (FEM), anisotropic porous medium, heat transfer, MAPLE, FEM

Introduction

Rotational thermal convection flows on bodies of axisymmetric geometry have been studied for a number of decades by engineers and mathematicians, initially due to interest in the aerospace sciences. The Coriolis forces experienced with rotation generated by the centrifugal field cause fluid to be impelled along the curved surface and substantially enhance heat transfer rates. Since the seminal review by Grief[1] interest in external rotating heat transfer has continued to flourish. These flows abound in chemical engineering processing where they arise in spinning cone distillation columns and centrifugal film evaporators[2], aeration devices and atomizers[3]. Many excellent experimental, theoretical and computational studies have been communicated to elucidate the interaction of viscous, rotational and other body force effects in such flows. Chamkha and Rashad[4] investigated unsteady heat and mass transfer due to MHD mixed convection flow past rotating vertical cone with chemical reaction and soret and dufour effects. Osalusi et al.[5] examined the effect of viscous dissipation and Joule heating on unsteady MHD flow on a rotating cone in rotating fluid. Narayana et al.[6] studied free magnetohydrodynamic flow and convection from a vertical spinning cone with cross-diffusion effects. Anilkumar and Roy[7] employed Bellman-Kalaba quasi-linearization and an implicit finite difference scheme to study transient heat and mass transfer from a rotating cone, computing Sherwood numbers for a range of rotation parameters, and observing that self-similar solutions are only admitted when both the free stream angular velocity and cone rotational velocity vary as linear inverse functions of time. Raju et al.[8] studied thermophoretic effect on double diffusive convective flow of a chemically

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reacting fluid over a rotating cone in porous. Further studies of rotational thermal convection from a spinning body have been presented by Ece[9] for steady magnetic convection.

The above studies have considered only pure fluid regimes external to the rotating body. In numerous energy resources areas, however the external medium is often porous i.e., comprises a permeable material. As such extra body forces must be incorporated into the analysis to account for linear porous resistance at lower Reynolds numbers. Porous media offer excellent properties for flow control and filtration. Most porous heat transfer flow studies have used the isotropic Darcy law[10,11], which assumes that permeability in all directions is the same. A more general case is that of anisotropic porous media[12] which incorporates a variation in the permeability depending on the direction. This better characterizes many synthetic and geological materials. Anisotropy is normally results of preferential orientation or asymmetric geometry of porous matrix or fibers. It occurs in many industrial system and nature. Besides, anisotropy can also be a characteristic of artificial porous materials such as pelleting used in chemical engineering process, fiber material used in insulating purpose and packed beds used in the storage of heat energy.

In the present study we shall therefore examine the thermal convection from a rotating cone to anisotropic Darcian porous media. This regime is relevant to filtration chemical engineering coating applications. A MAPLE numerical solution[13] to the transformed ordinary differential equations is obtained. The computations are validated with purely fluid (infinite permeability) solutions in the literature and also with a variational finite element code[14] based on linear elements. The present study presents a comprehensive examination of anisotropic permeability effects on rotating cone convection in porous media and to the authors' knowledge has not appeared in the literature thus far.

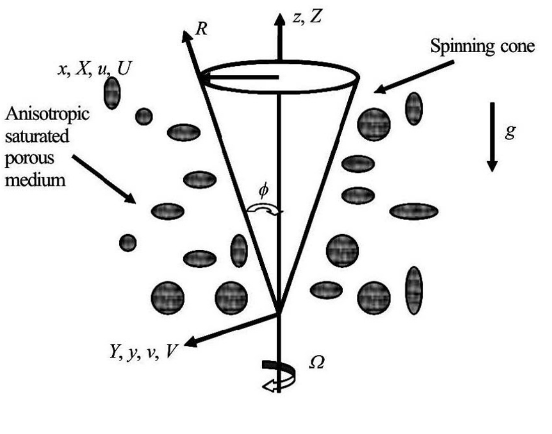

Fig.1 Physical model for convection from a rotating cone in an anisotropic porous regime

1. Mathematical rotating flow model

The physical model is depicted in Fig.1. We consider steady-state, laminar, incompressible, axisymmetric, free convection boundary layer flow along a rotating cone embedded in an anisotropic saturated porous regime.

Tortuosity and thermal dispersion effects in the porous medium are neglected. The Darcy model is employed[15]. The cone surface is isothermal. Rotation is sufficiently slow to ignored compressibility effects. The X direction is parallel to the cone slant surface, the Y direction normal to this and θ designates the angle in a plane perpendicular to the vertical symmetry axis. The cone may represent for example a chemical engineering mixing device. The governing equations for the flow regime can be posed as follows with reference to an (X, Y, θ) coordinate system:

Mass

$$\frac{\partial(RU)}{\partial X}+\frac{\partial(RV)}{\partial Y}=0 \tag{1}$$

Momentum

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}-\frac{W^2}{R}\frac{\mathrm{d}R}{\mathrm{d}X}=\nu\frac{\partial^2 U}{\partial Y^2}-\frac{\nu}{K_X}U+ g\beta\cos\alpha(T-T_\infty) \tag{2}$$

Momentum

$$U\frac{\partial W}{\partial X}+V\frac{\partial W}{\partial Y}+\frac{UW}{R}\frac{\mathrm{d}R}{\mathrm{d}X}=\nu\frac{\partial^2 W}{\partial Y^2}-\frac{\nu}{K_\theta}W \tag{3}$$

Energy

$$U\frac{\partial W}{\partial X}+V\frac{\partial W}{\partial Y}+\frac{UW}{R}\frac{\mathrm{d}R}{\mathrm{d}X}=\nu\frac{\partial^2 W}{\partial Y^2}-\frac{\nu}{K_\theta}W \tag{4}$$

The Boussinesq approximation has been used so that buoyancy effects only appear in the X-direction momentum Eq.(2), which is coupled to the energy equation, constituting a free convection regime. Using separate permeabilities in the X and θ directions (due to anisotropy), two porous media drag force terms are present, one in each of Eqs.(2) and (3) i.e., the primary and secondary Darcian impedance. Viscous dissipation and cross-diffusion (Soret/Dufour) effects are ignored. The corresponding boundary conditions at the surface and far from the cone are:

$$U(X,0)=V(X,0)=0\,,\ \ W(X,0)=\Omega R\,,$$

$$T(X,0)=T_\infty+(T_w-T_\infty)\frac{X}{L} \tag{5}$$