Available online at www.sciencedirect.com
ScienceDirect
Journal of Hydrodynamics
2016,28(2):219-226
DOI: 10.1016/S1001-6058(16)60623-6
www.sciencedirect.com/
science/journal/10016058

Three-dimensional unstructured modelling of wave-induced circulation over a plane and irregular beach*

Qiu-shun WANG (汪求顺)[1,2], Hai-gui KANG (康海贵)[2], Ke WANG (王科)[3]
1. Zhejiang Institute of Hydraulics and Estuary, Hangzhou 310020, China
2. State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian 116024, China, E-mail: wangqiushun57@163.com
3. State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology, Dalian 116024, China

(Received July 15, 2014, Revised January 23, 2015)

Abstract: Nearshore currents have a complicated circulation structure over a beach due to the mutual interaction between waves and currents. To investigate the wave-induced circulation over a beach, a three-dimensional unstructured model accounting for the combined actions of waves and currents is established. The wave distribution over the beach is computed by a wave model and the depth-dependent wave radiation stresses with the surface roller are employed in the hydrodynamic model. The present model takes the mixing coefficients and the bottom shear stress under waves and currents into account. To evaluate the three-dimensional unstructured model, the laboratory experiments over a plane and irregular beach are used to test the performance of the model. The undertow over a plane beach is well reproduced and the vertical variability is captured. The performance of the model over an irregular beach is well displayed in the reproduction of pairs of counter-rotating primary circulations at the embayment troughs. Meanwhile, the secondary circulations are observed in the swash zone. The model captures the circulation systems over a beach and the circulation structures of the wave-induced currents are well exhibited.

Key words: unstructured model, wave-induced currents, depth-dependent wave stress, circulation structures

Introduction

The wave-induced current is commonly seen in the coastal zone where the mutual interaction between the waves and currents is involved. In the formation of the wave-induced current, the undertow and the rip current would occur as the waves propagate over a beach. The undertow is the pronounced bottom current and moves seaward in the lower half of the water column while the rip current is narrow and has the relatively depth-uniform profile with a high offshore velocity. Their respective horizontal extents are the most distinguished difference between the rip current and the undertow. Due to the complicated flow pattern, there is a vertical circulation in the undertow and a horizontal circulation in the rip current. These circulations represent a mechanism for maintaining the mass balance and will affect the sediment transport in the nearshore zone. Consequently, the circulation structures of the wave-induced currents is an important issue for a beach.

In recent years, many laboratory experiments and numerical models were proposed for the study of wave-induced currents, especially the wave-induced rip currents[1-8]. Haller et al.[5] measured the horizontal velocity distribution of the rip currents on a barred beach. Drønen et al.[2] conducted an experiment to investigate the vertical characteristics of the rip currents, but the measurement of the velocity was limited to the surf zone with the swash zone largely ignored. In the previous experiment over the irregular beach, it was noticed that there were the secondary circulations around the swash zone[1,4,5]. But few numerical models can provide useful information about the secondary

---

* Project supported by the National Key Basic Research Development Program of China (973 Program, Grant No. 2013CB036101), the National Natural Science Foundation of China (Grant No. 51379037).

Biography: Qiu-shun WANG (1984-), Male, Ph. D., Engineer circulation. As the mechanism of the wave-induced nearshore currents is in a three-dimensional pattern, it is important to investigate the circulation system by a three-dimensional model.

In the three-dimensional models, the depth-dependent wave stress concept was proposed by some authors. Xia et al.[9] deduced a vertical variation wave stress by invoking the small amplitude approximation and applied it to the wave-current model, but it generates a reverse undertow in the surf zone, which is not consistent with the experimental results. Xie[7] established a three-dimensional structured grid model to investigate the wave-induced rip currents based on the depth-dependent wave stress concept proposed by Zhang[10]. With regards to the three-dimensional wave stresses, the formulation of Mellor[11] was widely used to study the wave-induced currents through various models[12,13]. Sheng and Liu[12] assessed the performance of the Mellor approach under realistic conditions, which improved the results of wave-induced currents. Kumar et al.[14] presented an implementation of the Mellor formulation into the structured grid model, including the changes in the vertical distribution of the wave radiation stress. Although the structured grid models play a pioneering role in studying the wave-induced currents, the unstructured models have an advantage in the irregular zone[13,15]. In view of the applications for the coastal waters, developing a three-dimensional unstructured model is desirable to study the wave-induced circulation over a beach.

This study aims to develop a three-dimensional wave-current model to resolve the circulation dynamics over a beach. Based on the three-dimensional circulation model finite volume coastal and ocean model (FVCOM), a wave-current model takes the mutual effect into account.

1. Model description

1.1 *Hydrodynamic model*

The three-dimensional equations involving the effect of waves are based on the circulation model FVCOM in the sigma (σ) coordinates[15], where the baroclinic terms are neglected and the horizontal diffusion terms are replaced with those in the present forms, and the wave radiation stress terms are added. The model is written as follows:

$$\frac{\partial \eta}{\partial t}+\frac{\partial (Du)}{\partial x}+\frac{\partial (Dv)}{\partial y}+\frac{\partial w}{\partial \sigma}=0 \tag{1}$$

$$\frac{\partial (uD)}{\partial t}+\frac{\partial (u^2 D)}{\partial x}+\frac{\partial (uvD)}{\partial y}+\frac{\partial (uw)}{\partial \sigma}=fvD-gD\frac{\partial \eta}{\partial x}+$$

$$\frac{\partial}{\partial x}\left(K_x D\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y D\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial \sigma}\left(\frac{K_z}{D}\frac{\partial u}{\partial \sigma}\right)-$$

$$\left\{\frac{\partial [DS_{xx}(\sigma)]}{\partial x}+\frac{\partial [DS_{xy}(\sigma)]}{\partial y}\right\} \tag{2}$$

$$\frac{\partial (vD)}{\partial t}+\frac{\partial (v^2 D)}{\partial y}+\frac{\partial (uvD)}{\partial x}+\frac{\partial (vw)}{\partial \sigma}=-fuD-gD\frac{\partial \eta}{\partial y}+$$

$$\frac{\partial}{\partial x}\left(K_x D\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y D\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial \sigma}\left(\frac{K_z}{D}\frac{\partial v}{\partial \sigma}\right)-$$

$$\left\{\frac{\partial [DS_{yx}(\sigma)]}{\partial x}+\frac{\partial [DS_{yy}(\sigma)]}{\partial y}\right\} \tag{3}$$

$$\frac{\partial p}{\partial \sigma}+\rho D g=0 \tag{4}$$

where u, v and w are the velocities in the x, y and σ directions, respectively, the vertical coordinate σ ranges from $\sigma=0$ at the free surface to $\sigma=-1$ at the bottom, t is the time, η is the free surface elevation, D is the total water depth, f is the Coriolis parameter, p is the pressure, ρ is the water density, g is the gravitational acceleration, K_x, K_y, K_z are the mixing coefficients under waves and currents in the x, y and σ directions, respectively. In the present study, the mixing coefficients depend on the bed shear velocities under waves and currents, which are relevant to the wave-current bottom shear stress. Hence, the horizontal mixing coefficients[16] are adopted as follows

$$K_x=K_y=\alpha_x u_* D \tag{5}$$

where $\alpha_x=5.93$, u_* is the bed frictional shear velocity due to the combined actions of waves and currents and $u_*=\sqrt{2\tau_{cw}/\rho}$, τ_{cw} is the bottom shear stress in the wave-current and is calculated according to a non-linear formula[17] as follows

$$\tau_{cw}=\tau_c\left[1+1.2\left(\frac{\tau_w}{\tau_c+\tau_w}\right)^{3.2}\right] \tag{6}$$

where τ_c is the current bottom shear stress, τ_w is the wave bottom shear stress,