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Applying the maximum information principle to cell transmission model of traffic flow*

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Abstract: This paper integrates the maximum information principle with the Cell Transmission Model (CTM) to formulate the velocity distribution evolution of vehicle traffic flow. The proposed discrete traffic kinetic model uses the cell transmission model to calculate the macroscopic variables of the vehicle transmission, and the maximum information principle to examine the velocity distribution in each cell. The velocity distribution based on maximum information principle is solved by the Lagrange multiplier method. The advantage of the proposed model is that it can simultaneously calculate the hydrodynamic variables and velocity distribution at the cell level. An example shows how the proposed model works. The proposed model is a hybrid traffic simulation model, which can be used to understand the self-organization phenomena in traffic flows and predict the traffic evolution.

Key words: kinetic traffic model, Cell Transmission Model (CTM), maximum information principle, traffic flow, velocity distribution

Introduction

The models for vehicular traffic flows can be divided into macroscopic, mesoscopic and microscopic ones. For the macroscopic model, the related variables are directly the velocity, density and flow flux. For the mesoscopic model, the main concern is the velocity distribution. For the microscopic model, the first concern is the microscopic driving behavior. In the present paper, we focus ourselves on the mesoscopic model.

Until now, the approaches to the evolution equations of velocity distribution can be summarized as three ones. The first approach is the Boltzmann-like treatments, which was initiated from a pioneered mesoscopic model. The second one is to use encounter rate and table of games, which is called the methods of discrete mathematical kinetic theory. The third one is to construct the lattice Boltzmann model of velocity distribution. The details can be referred to Ref.[1].

Vehicular traffic can be modeled as a system of interacting particles driven far from equilibrium. Using statistical physics methods to study vehicular traffic offers the possibility to examine various fundamental aspects of this kind of truly non-equilibrium systems[2]. Because phase transitions, hysteresis effects, and other nonlinear effects of synergetics determine spatiotemporal traffic pattern features, spatiotemporal traffic phenomena may be considered an aspect of synergetics[3,4]. Different classes of spatial-temporal patterns in traffic flows can be considered as different phases of the system. Kerner[5] modeled synergetic phenomena in spatial-temporal patterns as phase transition and defined a synchronized flow phase. Phase transitions in traffic flows on multilane

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roads were also studied in the framework of his three-phase traffic theory proposed by Kerner and Klenov[6,7].

Microscopic approach of synergetics has been applied to model jamming transition in traffic flows based on the Lorentz system. Synergetic scheme was proposed to describe the jamming transition in traffic flows, taking into account the internal fluctuations of characteristic acceleration/braking time[8]. On the basis of Ref.[8], the influence of the characteristic acceleration/braking time in the most probable headway deviation from its optimal value was studied, and headway deviation characterizing a phase transition was showed[9]. The homotopy perturbation method, the variational iteration method[10] and the differential transformation method[11] were used to give approximations to the governing equations offered in Refs.[8,9], and these three methods could provide highly accurate analytical solutions.

Recently, the macroscopic approach of synergetics has been used to model complex social systems. The integration of the macroscopic approach of synergetics and the continuity equation was used to model residential mobility macroscopically[12]. In this paper, we attempt to use the maximum information principle, a macroscopic approach of synergetics, to calculate the velocity distribution. The feature of this method is to use the macroscopic variables of traffic flow to derive the velocity distribution, without modeling the microscopic vehicle interactions.

1. Synergetics

Synergetics is initiated by Haken[13] in 1969, which dealt with complex systems, i.e., systems composed of many individual parts that are able to spontaneously form spatial, temporal or functional structures by means of self-organization. Synergetics has formed two theoretical branches: microscopic or mesoscopic approach and macroscopic approach. For the former approach, the concepts of instability, order parameters and slaving are used, which can be cast into a rigorous mathematical form, and one could show the emergence of structures and concomitantly of new qualities at a macroscopic level. For the latter approach[14], the maximum information principle is used, which is an analogy with thermodynamics. This approach treats complex systems by means of macroscopically observed quantities, and then determines the microscopic structure of the processes which give rise to the macroscopic structure. The maximum information principle claims that the probability distribution is the most possible probability distribution when the information is maximized. In this paper, we use the macroscopic approach to study the velocity distribution evolution of traffic flow.

2. Cell transmission model

The Lighthill-Whitham-Richards (LWR) model is a first-order hydrodynamic model of traffic flows, and a macroscopic approach that provides good approximation of traffic flow evolution in realistic networks. Many numerical methods have been developed to solve related problems with the LWR model. One approach is to solve the Riemann problem by applying the Godunov method. Another approach is to use the demand and supply functions[15,16], which turns out to be a variant of the Godunov method. The third approach is the wave tracking resolution scheme[17]. The Godunov discretization scheme is efficient as it has been proved that the flow is constant during a time step. The transmission flow can be easily calculated using the following formula

$$Q_x^t = \min\{S(K_{x-1}^t), R(K_x^t)\}$$

where S and R are the demand and supply functions, respectively defined by

$$S(K) = Q_E(k) \quad \text{if} \quad K < K_{\text{critical}},$$

$$S(K) = Q_{\max} \quad \text{if} \quad K \geq K_{\text{critical}}$$

$$R(K) = Q_{\max} \quad \text{if} \quad K < K_{\text{critical}},$$

$$R(K) = Q_E(k) \quad \text{if} \quad K \geq K_{\text{critical}}$$

With the transmission flow, we can write the density updating formula as

$$K(t+\Delta t, x) = K(t, x) - \frac{\Delta t}{\Delta x}(Q_{x+1}^t - Q_x^t)$$

The Cell Transmission Model (CTM) transforms the differential equations in the LWR model into simple difference equations. In the CTM, a road is divided into homogeneous and interconnected segments, referred to as cells, and piecewise linear relationships are assumed between flow and density at the cell level.

3. Proposed traffic kinetic model

Let flow flux and density for traffic flow be given. We wish we could derive the probability distribution of speed. In other words, we start from the macroscopic world and wish to draw conclusions about the microscopic world. In synergetics, a measure for the amount of information is connected with the number of possible events (realizations). There is an overwhelming probability of finding that the realized