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Mass transport in a thin layer of power-law fluid in an Eulerian coordinate system*

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Abstract: The mass transport velocity in a thin layer of muddy fluid is studied theoretically. The mud motion is driven by a periodic pressure load on the free surface, and the mud is described by a power-law model. Based on the key assumptions of the shallowness and the small deformation, a perturbation analysis is conducted up to the second order to find the mean Eulerian velocity in an Eulerian coordinate system. The numerical iteration method is adopted to solve these non-linear equations of the leading order. From the numerical results, both the first-order flow fields and the second-order mass transport velocities are examined. The verifications are made by comparing the numerical results with experimental results in the literature, and a good agreement is confirmed.

Key words: mass transport velocity, power-law model, periodic pressure load, Eulerian coordinates system

Introduction

The muddy estuarine coast is widely encountered around the world. The interaction between waves and soft mud is one of the key mechanisms controlling the transport of cohesive sediments in coastal and estuarine waters, and the mass transport generated by the wave motion is with a steady second-order drift velocity. Although its velocity is small in magnitude, the mass transport is one of the notable phenomena for the soft mud, because it plays an important role in determining the migration of nutrients and pollutants[1,2]. Moreover, it could also be used to predict the bed evolution in the long term.

Various models were proposed to describe the relationship between the stress and the shear rate for the soft mud. In most cases, the muddy bed is assumed as a linear medium, Liu and Chan[3], Hu et al.[4],Yang and Chen[5] and Ng[6] used a viscous fluid model to describe the deformable muddy bed, Lee et al.[7] used a poro-elastic model. Another linear model which combines the features of these two models was widely used, known as the visco-elastic solid model. The typical examples of this model include the Voigt model by Ng et al.[8] and the Jeffreys model by Niu and Yu[9]. Instead of the linear assumption, nonlinear models such as visco-plastic solid models were developed: Becker and Bercovici[10] and Zhang and Ng[11] adopted the Bingham plastic model, whereas Xia and zhu[12,13] used the Maxwell model to study the wave-mud interaction. Moreover, a visco-elasto-plastic model was adopted by Niu and Yu[14].

The studies mentioned above mainly focus on the wave damping up to the first-order. The mass transport velocity, which can be determined only when the second-order wave motion is considered, has received a great deal of attention in some theoretical and expermental studies. A general theory of the mass transport in water was developed by Longuet-Higgins[15]. Sakakiyama and Bijker[16] measured the mass transport velocity in the soft mud layer, with a progressive wave over a Bingham mud layer. The experimental results were compared with their theoretical results applicable only to a Newtonian fluid. Ng[17] developed an asymptotic theory in a thin bi-viscous mud layer, with

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the assumption that the mud layer thickness is comparable to the Stokes boundary layer thickness. Zhang and Ng[11] also deduced an analytical expression of the mass transport velocity in the mud layer, described by the viscoelastic Voigt model.

On the other hand, the Bingham model, with the mud being assumed as a rigid body at a low stress level and a fluid with a constant viscosity coefficient when the stress exceeds the yield stress, is widely used to describe the non-linear rheological properties of the soft mud. With this model, it is often difficult to determine the position of the yield surface in the transient theory, where the shear stress is equal to the yield stress, and the yield surface cannot be predefined and has to be resolved as a part of the problem. Therefore, a more convenient model which is known as the power-law model is preferred, it could describe the shear thinning phenomenon, especially at a low shear ratio. The power-law model could also well represent the non-linear rheological properties of the estuarine mud. This is verified by the experimental results of Ng et al.[8] and Tian[18] who studied the mud in the Haihe Estuary in China, similar results were obtained by Pang[19] who studied the mud in the Lianyun Harbor in China. Thus, the power-law model is used in this paper.

Under the action of a progressive wave, the pressure is transmitted from the water body to the mud layer, to produce the shear stress on the mud surface. Under the shear stress, the mud bed moves to form a muddy fluid with a high concentration. In this paper, a theory is developed to calculate the mass transport velocity in the layer of the non-Newtonian mud which behaves like a power-law fluid. To simplify the mathematical problem, unlike the traditional double-layer or multi-layer model, it is assumed that only a thin mud layer exists, a pressure load described as a periodic function of time and space is applied directly on the mud free surface, and it can reflect the wave action on the water-mud interface to a certain extent, and the similar assumption was adopted in the two-layer model by Bercovici and Becker[20]. In fact, the water viscosity is much smaller than the mud viscosity, the shearing action that the water imposes on the mud surface is not remarkable, so the shear stress on the free surface will be assumed to be equal to zero in this paper.

In a Lagrangian coordinate system, Ng[2] studied the mass transport in a thin layer of muddy fluid, and a straightforward expression of the mass transport velocity was induced. However, the derived equations were very complex and non-intuitive. Thus, the theory in this paper will be based on an Eulerian coordinate system. Different from the approach of Sakakiyama and Bijker[16], the mass transport velocity investigated by Longuet-Higgins[15] is adopted in this paper so that the mechanism of the mud mass transport could be fully described.

1. Analytical model and method

1.1 *Mathematical formulation of the problem*

Consider a single uniform mud layer lying on the horizontal rigid bed with an undisturbed depth h, the mud's behavior is described by a power-law model with a high concentration. The mud layer is homogeneous and incompressible with a constant density ρ, and the size of the mud layer is assumed to be infinite in the longitudinal direction., an external periodic pressure load is applied directly on the mud surface to drive the wave motion, and it is a periodic function of time and space with a form like the periodic progressive wave: $p=-p_s\cos(kx-\sigma t)$, p_s is the amplitude, k is the wave number, and σ is the angular frequency. The mud wave propagates along the x direction, the bottom of the mud layer is fixed at $z=-h$, and the elevation of the free surface $z=0$ is denoted by $\eta=(x,0,t)$. The sketch of the problem geometry is shown in Fig.1.

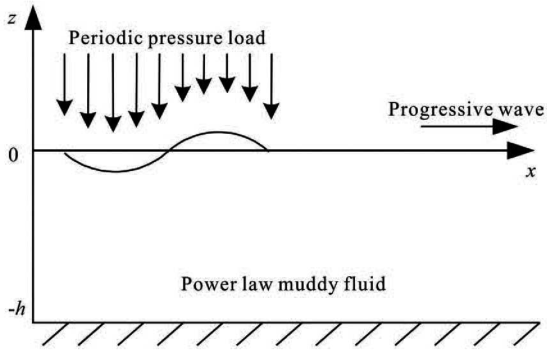

Fig.1 Sketch of the problem geometry

The motion equations, which include the continuity equation for an incompressible fluid and the momentum equations in horizontal and vertical planes, are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+\frac{\partial (uu)}{\partial x}+\frac{\partial (uw)}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xz}}{\partial z}\right) \tag{2}$$

$$\frac{\partial w}{\partial t}+\frac{\partial (uw)}{\partial x}+\frac{\partial (ww)}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+\frac{1}{\rho}\left(\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{zz}}{\partial z}\right)-g \tag{3}$$

where x and z are the horizontal and vertical coordinates, respectively, u and w are the horizontal and