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Numerical study of the flow in the Yellow River with non-monotonous banks*

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Abstract: A 2-D depth averaged RNG k - ε model is developed to simulate the flow in a typical reach of the Upper Yellow River with non-monotonic banks. In order to take account of the effect of the secondary flow in a bend, the momentum equations are modified by adding an additional source term. A comparison between the numerical simulation and the field measurements indicates that the improved 2-D depth averaged RNG k - ε model can improve the accuracy of the numerical simulation. An arc spline interpolation method is developed to interpolate the non-monotonic river banks. The method can also be reasonably applied for the 2-D interpolation of the river bed level. Through a comparison of the water surface gradients simulated in the seven bends of the studied reach, some analytical formulae are improved to reasonably calculate the longitudinal and transverse gradients in meandering river reaches. Furthermore, the positions of the maximum water depth and the maximum velocity in a typical bend are discussed.

Key words: 2-D RNG k - ε model, numerical simulation, extra source term, arc cubic spline interpolation, non-monotonic bank

Introduction

The flow structures in rivers with bends are very important in river engineering as they are closely related to the river flooding prediction, the bank protection, the water intakes, the navigation, and the evolution of the river-bed and the sediment transport. As almost all natural rivers are meandering, the study of the flow structures in meandering rivers/channels is of enormous practical importance in engineering applications. Therefore, in past decades, the flow structures in meandering rivers/channels were extensively studied using various approaches, such as the theoretical analysis, the laboratory experiments and the numerical simulation.

In a theoretical analysis, some theoretical solutions or empirical formulae are developed to calculate the flow structure in meandering rivers/channels[1]. However, they more often than not contain some simplifications of the flow conditions, and some are obtained by analyzing the measured data. Therefore, they may not be valid for all flow conditions. The laboratory experiment is a very important approach to study the flow structures in meandering rivers[2,3]. However, expensive economical and time investments are involved. With the development of the computational methods and computer techniques, numerical simulations are increasingly applied for the simulation of the flow in rivers and channels. In past decades, many methods were developed and used to simulate the turbulent flow in rivers or open channels, such as the methods based on the potential flow theory[4], the direct numerical simulation (DNS)[5], the large eddy simulation (LES)[6,7], and the Reynolds averaged numerical simulation (RANS)[8,9]. Though the DNS and the LES are more accurate, their huge computation requirement limits their applications in the numerical simulation of

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natural rivers. Compared with the DNS and the LES, the RANS needs less computation time and is widely applied in numerical simulations of flows in rivers/ channels. For the RANS, we have the zero equation model, the k - ε model, the Reynolds stress equation model (RSM), and the algebraic stress model (ASM), etc.[10,11]. Among these turbulent models, the k - ε model is more adaptive than the zero equation model and it requires less computation time than the RSM and the ASM. As a result, it is widely used to numerically simulate all kinds of flows, such as open channel flows, river flows and pipe flows[12,13]. However, the standard k - ε model may produce distorted results when it is used to simulate some complex flows, such as the flow with a high strain rate, and the flow with a large streamline curvature. For the flow with a high strain rate and a large streamline curvature, the renormalization group k - ε model (RNG k - ε model) is applied in order to obtain accurate simulation results.

The flow in meandering rivers is 3-D and should be simulated by 3-D turbulence models. However, because of the long computation time and the storage requirement for the computer, 3-D turbulence models are difficult to be applied to simulate the flow and the sediment transport in a long river reach for long time. Therefore, the depth-averaged and the 2-D depth averaged mathematical models were proposed. Generally speaking, 2-D models can not reflect the influence of the secondary flow in meandering rivers, and the simulated results are usually not satisfactory. In order to solve this problem, various schemes were proposed in recent years. Lien et al.[14] developed a 2-D model with consideration of the influence of the secondary flow through calculating the dispersion stress produced by the discrepancy between the mean and the true velocity distributions. Kimura et al.[15] applied a 2-D depth averaged model to simulate the open channel flow in a side-cavity, in which the effects of the secondary flow are integrated by adding an extra source term in the momentum equations. However, the above models can only be used in laboratory regular meandering channels. In the above models, the additional source terms in the momentum equations are very complicated, and the governing equations are usually formulated in orthogonal coordinates, in which the velocity components are not natural. In addition, most of these models use laminar flow models or zero equation models. Therefore, they are not easy to be applied to numerically simulate the turbulent flow in meandering natural rivers.

In this study, a 2-D depth averaged RNG k - ε model is developed by averaging the 3-D RNG k - ε model in depth and by adding an extra source term of the momentum equations in the body-fitted coordinate system. The model is applied to simulate the turbulent flow in the Daliushu Reach of the Yellow River. Simulated velocity and water level agree well with the field measured data. The water surface slopes along the longitudinal and transverse directions in typical bends of the studied river reach are simulated and analyzed. The distributions of the turbulence kinetic energy and the turbulence dissipation rate at some typical cross-sections are also discussed. Numerical examples indicate that the improved 2-D RNG k - ε model can reasonably simulate the turbulent flow in rivers with monotonic banks.

Before simulation, some preprocessing work is required, such as the river bank interpolation, the topography interpolation, and the mesh generation. Because the boundary of the simulated physical area is non-monotonic, the most effective interpolation method, i.e. the cubic spline interpolation, becomes invalid for the bank interpolation in the studied reach. In addition, the ordinary 2-D interpolation methods also do not work when they are used for the topography interpolation. In order to solve these problems, a new method based on the cubic spline interpolation, named as the arc cubic spline interpolation, is developed for the bank and topography interpolations in this study. Examples show that the arc cubic spline interpolation is very efficient when it is used for the non-monotonic river bank interpolation and the 2-D topography interpolation. The improved RNG k - ε model, incorporated with the sediment model, can also be applied to simulate the sediment transport and the bed deformation in rivers or open channels. The simulated results will be presented in subsequent papers.

1. Mathematical model

In a body-fitted coordinate (BFC) system, five governing equations of the 2-D RNG k - ε model can be written in the following universal form[16,17]

$$\frac{\partial}{\partial t}(h\Phi)+\frac{1}{J}\frac{\partial}{\partial \xi}(hU\Phi)+\frac{1}{J}\frac{\partial}{\partial \eta}(hV\Phi)=$$

$$\frac{1}{J}\frac{\partial}{\partial \xi}\left(\frac{\alpha h\Gamma_{\Phi}}{J}\frac{\partial \Phi}{\partial \xi}\right)+$$

$$\frac{1}{J}\frac{\partial}{\partial \eta}\left(\frac{\gamma h\Gamma_{\Phi}}{J}\frac{\partial \Phi}{\partial \eta}\right)+S_{\Phi}(\xi,\eta) \qquad (1)$$

where Φ is the universal variable, Γ_{Φ} is the diffusion coefficient and S_{Φ} is the source term are shown in Table 1.