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Theoretical analysis of slip flow on a rotating cone with viscous dissipation effects*

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Abstract: This paper is concerned with the mutual effects of viscous dissipation and slip effects on a rotating vertical cone in a viscous fluid. Similarity solutions for rotating cone with wall temperature boundary conditions provides a system of nonlinear ordinary differential equations which have been treated by optimal homotopy analysis method (OHAM). The obtained analytical results in comparison with the numerical ones show a noteworthy accuracy for a special case. Effects for the velocities and temperature are revealed graphically and the tabulated values of the surface shear stresses and the heat transfer rate are entered in tables. From the study it is seen that the slip parameter γ enhances the primary velocity while the secondary velocity reduces. Further it is observed that the heat transfer rate $NuRe_x^{-1/2}$ increases with Eckert number Ec and Prandtl number Pr.

key words: mixed convection, incompressible flow, differential equations, slip effects, viscous dissipation

Introduction

A study which involves the equivalent participation of both forced and natural convection is termed as mixed convection. It plays a key role in atmospheric boundary layer flows, heat exchangers, solar collectors, nuclear reactors and in electronic equipment's. Such processes occur when the effects of buoyancy forces in forced convection or the effects of forced flow in natural convection become much more remarkable. The interaction of both convections is mostly noticeable in physical situations where the forced convection flow has low velocity or moderate and large temperature differences. In the concerned analysis, a rotating cone is placed in a Newtonian fluid with the axis of the cone being in line with the external flow is inspected. The mixed convective heat transfer problems with cones are generally used by automobile and chemical industries. Some important applications are design of canisters for nuclear waste disposal, nuclear reactor cooling system, etc.. Practically, the unsteady mixed convective flows do not give similarity solutions and for the last few years, various problems have been deliberated, where the non-similarities are taken into account. The unsteadiness and non-similarity in such type of flows is due to the free stream velocity, the body curvature, the surface mass transfer or even possibly due to all these effects. The crucial mathematical difficulties elaborate in finding non-similar solutions for such studies have bounded several researchers to confine their studies either to the steady non-similar flows or to the unsteady semi-similar or self-similar flows. A solution is recognized as self-similar if a system of partial differential equations can be reduced to a system of ordinary differential equations. If the similarity transformations are able to reduce the number of independent variables only, then the reduced equations are named semi-similar and the corresponding solutions are the semi-similar solutions. Hering and Grosh[1] studied steady mixed convection boundary layer flow from a vertical cone in an ambient fluid for the Prandtl number of air. Himasekhar et al.[2] carried out the similarity solution of the mixed convection boundary layer flow over a vertical rota-

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ting cone in an ambient fluid for a wide range of Prandtl numbers. A few years back, Anilkumar and Roy[3] obtained the self-similar solutions of unsteady mixed convection flow from a rotating cone in a rotating fluid. Unsteady heat and mass transfer from a rotating vertical cone with a magnetic field and heat generation or absorption effects were examined by Chamkha and Al-Mudhaf[4].The non-similar solution to study the effects of mass transfer (suction/injection) on the steady mixed convection boundary layer flow over a vertical permeable cone were presented by Ravindran et al.[5]. Also Nadeem and Saleem[6] explore the analytical study of mixed convection flow of non-Newtonian fluid on a rotating cone. Hall effects on unsteady flow due to noncoaxially rotating disk and a fluid at infinity were presented by Hayat et. al.[7]. Fluids revealing slip are significant in technological applications such as in the polishing of artificial heart valves and internal cavities. Slip also occurs on hydrophobic surfaces, particularly in micro- and nano-fluidics. Makinde and Osalsui[8] studies MHD steady flow in a channel with slip at the permeable boundaries. Ellahi et. al.[9] examined the study of generalized Couette flow of a third grade fluid with slip: the exact solutions. Some relevant studies on this phenomenon are given in Refs.[10-15]. The influence of variable viscosity and viscous dissipation on the non-Newtonian flow was explored by Hayat et. al.[16].

In general it is challenging to handle nonlinear problems, especially in an analytical way. Perturbation techniques like variation of iteration method (VIM) and homotopy perturbation method (HPM)[17,18] were frequently used to get solutions of such mathematical investigation. These techniques are dependent on the small/large constraints, the supposed perturbation quantity. Unfortunately, many nonlinear physical situations in real life do not always have such nature of perturbation parameters. Additional, both of the perturbation techniques themselves cannot give a modest approach in order to adjust or control the region and rate of convergence series. Liao[19] presented an influential analytic technique to solve the nonlinear problems, explicitly the homotopy analysis method (HAM). It offers a suitable approach to control and regulate the convergence region and rate of approximation series, once required.

Encouraged by all above findings, the main emphasis of the present paper is to examine the effects of slip on boundary layer flow over a rotating cone in a viscous fluid with viscous dissipation. The concerned nonlinear partial differential for rotating cone are transformed to system of nonlinear ordinary differential equations with proper similarity transformations and then solved by optimal homotopy analysis method (OHAM)[19-29]. Also the effects of related physical parameters on velocities, surface stress tensors, temperature and heat transfer rate are reported and discussed through graphs and tables.

1. Analysis of the problem

Consider the unsteady, axi-symmetric, incompressible viscous fluid flow of over a rotating cone in a Newtonian fluid. It is assumed that only the cone is in rotation with angular velocity which is a function of time. This develops unsteadiness in the flow field. Rectangular curvilinear coordinate system is taken to be fixed. Here u, v and w be the components of velocity in x, y and z-directions, respectively. The temperature as well as concentration variations in the flow fluid are responsible for the existence of the buoyancy forces. The gravity g acts downward in the direction of axis of the cone. Moreover, the wall temperature T_w and wall concentration C_w are linear functions of x, while the temperature T_∞ and concentration C_∞ far away from the cone surface are taken to be constant. The physical model and coordinate system is shown in Fig.1.

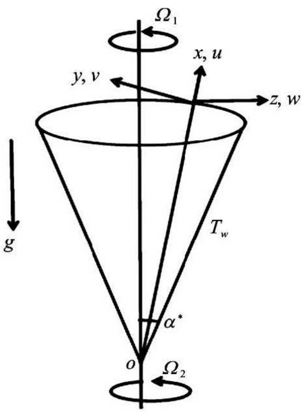

Fig.1 Physical model and coordinate system

By using Boussinesq approximation and boundary layer theory, the governing momentum and energy equations are deliberated as:

$$\frac{\partial(xu)}{\partial x}+\frac{\partial(xw)}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}-\frac{v^2}{x}=\nu\frac{\partial^2 u}{\partial z^2}+g\beta\cos\alpha^*(T-T_\infty) \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+w\frac{\partial v}{\partial z}+\frac{uv}{x}=\nu\frac{\partial^2 v}{\partial z^2} \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}=\alpha\frac{\partial^2 T}{\partial z^2}+\frac{\mu}{\rho C_p}\left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right] \tag{4}$$