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Ski jump trajectory with consideration of air resistance*

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Abstract: In the case of the ski-jump type energy dissipation, the jet trajectory will be greatly affected by the air entrainment and the air resistance. It is necessary to consider those factors when estimating the trajectory of the jet flow. In this work, the effect of the air resistance on the jet trajectory is theoretically and experimentally investigated. A comprehensive resistance coefficient is proposed. To determine this coefficient, experiments of five models are conducted with the circular-shaped flip bucket placed at the point of the takeoff of ski jumps. It is shown that, this coefficient of the lower jet trajectory is only related to the approach flow Froude number, while that of the upper jet trajectory is dominated by both this Froude number and the deflection angle. Furthermore, the present methodology is validated by experimental data in this work and the maximum errors are not larger than 3.2% and 8.6% for the lower and upper jet trajectories, respectively.

Key words: trajectory, jet flow, comprehensive resistance coefficient, hydropower project

The jet flow trajectory through a ski-jump is an important issue in the designs with respect to the ski-jump type energy dissipation. In practical projects, the jet flow will split and the air will be entrained into the flow when it leaves from the flip bucket, and the trajectory will be greatly affected by the air entrainment and the air resistance. It is necessary to consider those factors when estimating the trajectory of the jet flow.

Figure 1 shows a plane ski-jump flow with related notation definitions. The origin of the coordinate system (x, y) is at the edge of the flip bucket. In this figure, V_o and h_o are the approach flow velocity and depth, respectively, at the acting water head H_o, R_o and β are the radius and the deflection angle of the circular-shaped flip bucket, respectively, α_L and α_U are the virtual take-off angles of both the lower (subscript L) and upper (subscript U) jet trajectories, x_L and x_U are the impact points of the jet onto the tailwater channel, determined visually from the channel side, by extending the jet trajectories, and s is the height from the edge of the flip bucket to the tailwater channel bottom. The objectives of this paper are to theoretically and experimentally investigate the effects of the air resistance on the jet trajectory, and to determine these influencing factors.

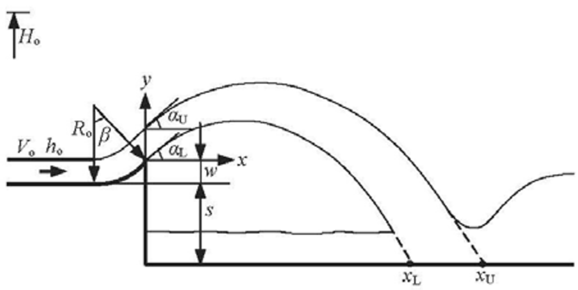

Fig.1 A plane ski-jump flow with related notation definitions

On the basis of the theory of the projectile for a rigid body[1], the trajectory of a mass point without

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consideration of the air resistance F_A can be expressed as

$$y = x\tan\beta - \frac{gx^2}{2(V_o\cos\beta)^2} \tag{1}$$

where $g = 9.81\,\mathrm{m^2/s}$, is the acceleration of the gravity.

For the jet flow from the flip bucket (Fig.1), the trajectory under the action of the air resistance is

$$y = x\tan\beta - K\frac{gx^2}{2(V_o\cos\beta)^2} \tag{2}$$

where K is called the comprehensive resistance coefficient. Liu and Zhang theoretically presented an expression of K, but there are some variables in the expression that cannot be obtained, including the density ratio of air (ρ_a) and jet flow (ρ_w), and the resistance coefficient (c_f). Using the field data of eight projects about the location of the deepest scour, they decided that K is only a function of the approach flow Froude number (Fr_o) [2]. In fact, K might be related to the geometric parameters of the flip bucket, and the hydraulic parameters of the approach flow and the jet flow. For a circular-shaped flip bucket, K could be constructed as (Fig.1)

$$K = f(V_o, h_o, g, R_o, \beta, \alpha_U, \alpha_L, C, c_f) \tag{3}$$

where C is the average air concentration of the flow section, $c_f = F_A/\rho_a A V_o^2$, the resistance coefficient of the air towards the flow, and A is the section area of the flow. Equation (3) could be rewritten in view of dimensional analysis as

$$K = f\left(Fr_o, \frac{h_o}{R_o}, \beta, \alpha_U, \alpha_L, C, c_f\right) \tag{4}$$

where $Fr_o = V_o/(gh_o)^{1/2}$, is the approach flow Froude number, h_o/R_o is the relative depth of the flow. Equation (4) could be further simplified. Firstly, the lower and upper take-off angles (α_L) and (α_U) of the flow are all related to the deflection angle (β) and the radius (R_o) of the circular-shaped flip bucket, and the depth (h_o) of the approach flow[3]. Meanwhile, they are also affected by the transverse fluctuating velocity (u') [4], given by

$$u' = 1.36\frac{nV_o}{R_H^{1/6}}\sqrt{g} \tag{5}$$

where n is the Manning roughness coefficient, and R_H is the hydraulic radius. Then we have

$$\alpha_U, \alpha_L = f_1(V_o, h_o, g, R_o, \beta, n, R_H) \tag{6}$$

While R_H is equal approximately to h_o due to the small flow depth relative to the width of the flip bucket, then Eq.(6) becomes

$$\alpha_U, \alpha_L = f_1\left(Fr_o, \frac{h_o}{R_o}, \beta, n\right) \tag{7}$$

Secondly, the air concentration (C) of the flow is dominated by the relative depth of the flow (h_o/R_o) and the approach flow Froude number (Fr_o) if the air density (ρ_a) is considered as a constant[5], thus,

$$C = f_2\left(Fr_o, \frac{h_o}{R_o}\right) \tag{8}$$

At the same time, the resistance coefficient (c_f) is related with the section form of the flow, and depends on h_o/R_o and Fr_o. Therefore, c_f could be written as

$$c_f = f_3\left(Fr_o, \frac{h_o}{R_o}\right) \tag{9}$$

On the basis of Eqs.(7)-(9), Eq.(4) could be simplified as

$$K = f\left(Fr_o, \frac{h_o}{R_o}, \beta, n\right) \tag{10}$$

For the flip bucket of the same material, the Manning roughness coefficient (n) is equal to a constant. Neglecting n, Eq.(10) could finally be rewritten as

$$K = f\left(Fr_o, \frac{h_o}{R_o}, \beta\right) \tag{11}$$

This implies that the comprehensive resistance coefficient (K) is a function of Fr_o, h_o/R_o and β. With an appropriate experimental procedure, the variations of the variable parameters in the comprehensive resistance coefficient in Eq.(11) can be determined to find the effect of each parameter separately on K.

The experiments are conducted in the High-speed