Available online at www.sciencedirect.com
ScienceDirect
Journal of Hydrodynamics
2014,26(3):416-423
DOI: 10.1016/S1001-6058(14)60047-0
www.sciencedirect.com/
science/journal/10016058

The mechanical energy equation for total flow in open channels*

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(Received June 30, 2013, Revised March 16, 2014)

Abstract: The mechanical energy equation is a fundamental equation of a 1-D mathematical model in Hydraulics and Engineering Fluid Mechanics. This equation for the total flow used to be deduced by extending the Bernoulli's equation for the ideal fluid in the streamline to a stream tube, and then revised by considering the viscous effect and integrated on the cross section. This derivation is not rigorous and the effect of turbulence is not considered. In this paper, the energy equation for the total flow is derived by using the Navier-Stokes equations in Fluid Mechanics, the results are as follows: (1) A new energy equation for steady channel flows of incompressible homogeneous liquid is obtained, which includes the variation of the turbulent kinetic energy along the channel, the formula for the mechanical energy loss of the total flow can be determined directly in the deduction process. (2) The theoretical solution of the velocity field for laminar flows in a rectangular open channel is obtained and the mechanical energy loss in the energy equation is calculated. The variations of the coefficient of the mechanical energy loss against the Reynolds number and the width-depth ratio are obtained. (3) The turbulent flow in a rectangular open channel is simulated using 3-D Reynolds averaged equations closed by the Reynolds stress model (RSM), and the variations of the coefficient of the mechanical energy loss against the Reynolds number and the width-depth ratio are discussed.

Key words: open channel, mechanical energy equation, steady flow, turbulent flow

Introduction

The continuity equation, the momentum equation and the mechanical energy equation of the total flow are the basis of the 1-D flow description in Hydraulics and Engineering Fluid Mechanics[1], and the mechanical energy equation is particularly a fundamental part. In the field of Fluid Mechanics, focuses are on the mechanism of the turbulent flow[2-5], the flow stability[6-8] and the related spatial or temporal distributions of the flow characteristics, studied by experiments or numerical simulations. For example, Abusbeaa and Shmela[9] conducted experiments over smooth channel beds with laser Doppler anemometer (LDA) to investigate the mean velocity distribution of uniform turbulent flows and compared these profiles with some available models. Nezu and Onitsuka[10] studied turbulent structures in partly vegetated open-channel flows with LDA and particle image velocimetry (PIV). Sanjou et al.[11] studied the turbulence structure in compound open-channel flows with one-line emergent vegetation by acoustic Doppler velocimeter (ADV). Dong and Zheng[12] studied spiral turbulence with direct numerical simulation (DNS). Chung and Pullin[13] investigated stationary buoyancy-driven turbulence with DNS and large-eddy simulation (LES). However, the energy equation and the mechanical energy loss were not well studied based on the fundamental theory in Hydraulics and Fluid Mechanics.

The energy equation for the total flow (the integral form of the energy equation) used to be deduced in the following manner in Hydraulics: (1) Constructing a stream tube around a stream line and extending the Bernoulli's equation[1] for the ideal fluid along the stream line to the stream tube to obtain the energy equation for the stream tube. (2) Adding a term to consider the viscous effect in the energy equation for the stream tube, and integrating this equation in the cross section to obtain the energy equation for the total real fluid. The integration form of the adding term is called the energy loss (the mechanical energy loss) of the total flow which should be determined by experimental or observational data. This energy equa-

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tion is still being used. However, there are shortcomings from the point of hydrodynamics: (1) The variations of the turbulent kinetic energy along the main flow direction can not be described. (2) The assumption of a static pressure distribution at the cross section can not be justifiable for the 3-D turbulent flow since there exists secondary flows at the cross section. (3) The formula of the mechanical energy loss can not be determined directly. Although the transformation and the loss of the mechanical energy for steady incompressible pipe flows of homogeneous fluid were studied in Ref.[14], the energy equation for the total flow in the open channel has not be obtained yet. Therefore, in this paper, a new energy equation for steady incompressible channel flows of homogeneous liquid is derived to avoid the shortcomings mentioned above. In addition, the velocity distributions of steady laminar and turbulent flows in a rectangular channel are obtained analytically or numerically, and the mechanical energy loss in the energy equation is studied based on these results.

1. Formulation of the mechanical energy equation

In general, there are differences between the open-channel flow and the pipe flow, which may be summarized as follows: (1) The open-channel flow is a gravity-driven flow while the pipe flow may be driven by both the pressure difference and the gravity. (2) The boundaries of the pipe flow are solid boundaries, while there exist both solid boundaries and free surfaces in the open channel flow, with different boundary conditions. Consider the control volume V in the open channel flow as shown in Fig.1. The surface A of the control volume V is composed of two cross-sections A_1, A_2 at the upstream and the downstream with a distance L between them, the channel wall and the free surface, and the angle between x_1 axis and the horizontal direction is θ. As shown in Fig.1, suppose that the equation for the free surface under the condition of a steady flow is $\eta = \eta(x_1, x_2)$, the components n_i of the unit normal vector to the free surface pointing in the external direction are

$$n_i = \left(\frac{\frac{\partial \eta}{\partial x_1}}{E}, \frac{\frac{\partial \eta}{\partial x_2}}{E}, \frac{-1}{E} \right) \tag{1}$$

where

$$E = \sqrt{1 + \left(\frac{\partial \eta}{\partial x_1}\right)^2 + \left(\frac{\partial \eta}{\partial x_2}\right)^2}$$

In addition, based on the kinematical boundary conditions of the free surface, the vertical velocity $u_3|_\eta$ on the free surface is

$$u_3|_\eta = u_1|_\eta \frac{\partial \eta}{\partial x_1} + u_2|_\eta \frac{\partial \eta}{\partial x_2} \tag{2}$$

Combining Eq.(1) with Eq.(2), we have $u_i|_\eta n_i =$ 0 at the free surface for a steady flow. In addition, for the open channel flow, the velocity differences between the liquid and the air is usually not so large, and the curvature radius of the free surface is large enough that the effect of the surface tension can be ignored, therefore, we may assume that there is no shear stress on the free surface and the normal stress is just the atmosphere pressure.

The differential form of the (mechanical) energy equation for a steady flow of incompressible homogeneous liquid in the gravitational field can be obtained using the Navier-Stokes equations, which is

$$\frac{\partial}{\partial x_i}\left[\rho u_i \left(g x_3 \cos\theta + \frac{p}{\rho} + \frac{1}{2} u_j u_j \right)\right] = \frac{\partial(\tau_{ij} u_i)}{\partial x_i} - \tau_{ij} s_{ij} + \frac{\partial}{\partial x_i}(\rho u_i g x_1 \sin\theta) \tag{3}$$

where ρ is the liquid density and u_i, p, τ_{ij}, s_{ij} are the velocity, the pressure, the viscous stress and the rate of deformation, respectively. The integral form of the energy equation for steady flows in laminar and turbulent states (corresponding to the statistical quantities in ensemble average) will be discussed below based on Eq.(3).

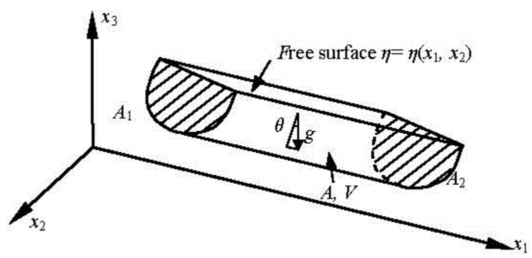

Fig.1 Sketch of open channel flow

1.1 *Laminar open channel flow*

Integrating Eq.(3) over the control volume V shown in Fig.1, and using the Gaussian Theorem to transform the volume integral into the surface integral for the left term and the first and third terms on the right hand side, we have