Available online at www.sciencedirect.com
ScienceDirect
Journal of Hydrodynamics
2014,26(3):431-435
DOI: 10.1016/S1001-6058(14)60049-4
www.sciencedirect.com/science/journal/l0016058

A numerical investigation of the flow between rotating conical cylinders of two different configurations*

LI Xue (李雪), ZHANG Jing-jing (张晶晶), XU Lan-xi (许兰喜)
Department of Mathematics, Beijing University of Chemical Technology, Beijing 100029, China,
E-mail: janemath@126.com

(Received December 11, 2012, Revised March 3, 2014)

Abstract: The flow between two coaxial conical cylinders is numerically studied for two different configurations, with the inner cone rotating and the outer one at rest. It is found that, in one configuration,at least at a small Reynolds number (Re), the pressure is a decreasing function of z while in the other configuration, it is an increasing function of z. In the first configuration, the pressure curves for different Re have intersections, while in the second configuration they do not. The gap between two conical cylinders is filled with six pairs of Taylor vortices at about the same Reynolds number and in each pair of vortices in the first configuration, the upper vortex is larger than the bottom one while in the second configuration, the bottom vortex is larger than the upper one.

Key words: rotating conical cylinder, Taylor vortex, Reynolds number, pressure distribution

Introduction

The flow between two concentric cylinders commonly referred to as the Taylor Couette flow is one of the most studied problem in fluid mechanics. It is a classical system used to investigate properties of flow driven by rotation. The results were numerous (see Refs.[1-7] and the references therein). The major interests were focussed on the occurrence of toroidal cells known as Taylor vortices.

The Taylor vortices may also occur in geometries other than between right circular cylinders, e.g., between rotating conical cylinders. In the last two decades, the Taylor vortices in the flow between two coaxial conical cylinders with inner cone rotating and outer one at rest were studied both experimentally and numerically. Wimmer[8] experimentally investigated the occurrence of the Taylor vortices and the influence of the governing parameters on the Taylor vortices. He showed that the laminar basic flow is three-dimensional. In Ref.[9], the transition to the turbulence was considered for the flow in a concentric annulus formed by conical cylinders of the same apex angle. The stability of the helical flow in the configuration of Fig.1(b) was experimentally investigated in Ref.[10], and it was found that the helical flow was resulted from a Hopf bifurcation. Noui-Mehidi et al.[11] studied the effect of the cylinder's wall alignment on the flow. He also considered the bifurcations of the steady vortical structures in the case when the cylindrical walls are not perfectly parallel. Xu et al.[12] showed that the behavior of the flow is dominated by a competition between the meridional flow and the radial flow. It was found that the vortices occur in the direction toward smaller radius. The local minimum values of the velocity and the local maximum values of the pressure are attained at the same point whereas the velocity of the flow takes the local maximum at the point of the inflection for the pressure. Altmeyer et al.[13] studied the effects of the end wall on the transition between the Taylor vortices and the spiral vortices. Zhang et al.[14] analyzed the effects of the end plates on the flow in the configuration of Fig.1(b) and noted that the Taylor vortices filled the gap are in an odd number when the inner cone rotates together with the top end plate, whereas they are always in an even number in

* **Biography:** LI Xue (1979-), Female, Ph. D. Candidate
Corresponding author: XU Lan-xi,
E-mail: xulx@mail.buct.edu.cn

our case. However, all previous studies were mainly related to the configuration of Fig.1(b). At the same time, the flows in the configuration of Fig.1(a) were not given a due attention.

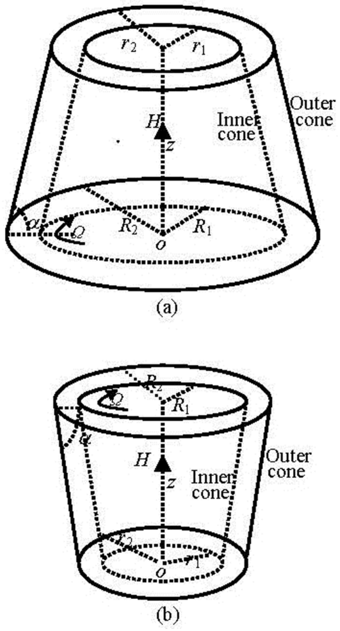

Fig.1 The sketch of two different configurations of coaxial rotating conical cylinders

Recently, the configuration of Fig.1(a) has caught our interest. A few chemists used the configuretion of Fig.1(a) as a precipitation reactor called the rotating liquid film reactor (RLFR), as the reactor to prepare new functional nano-particles. The gap between two cones is filled with reactants, which are usually considered as a viscous incompressible fluid. It is found that the particles produced in the RLFR are smaller in size and more concentrated in the size distribution, compared with the conventional precipitation reactors[15]. In order to understand the effect of the RLFR on the precipitation, it is necessary to investigate the flow properties in the gap. In order to do that, the configuration of Fig.1(a) should be considered and the flow property should be compared with that in Fig.1(b), which has motivated the study of this paper.

1. Mathematical formulation

The two different configurations are shown in Fig.1, in which the configuration in Fig.1(b) is just that in Fig.1(a) but upside down. The gap between two cones is filled with a viscous incompressible fluid. The inner cone rotates at the angular velocity Ω and the outer one is at rest. For both configurations, we assume that the top and bottom end plates are rigid and the cone's wall is a no-slip boundary. Then the governing equations (the Navier-Stokes equations) and the boundary conditions are as follows:

$$\partial_t \boldsymbol{u} = \nu \Delta \boldsymbol{u} - \frac{1}{\rho}\nabla P - \boldsymbol{u}\cdot\nabla\boldsymbol{u}\ , \quad \nabla\cdot\boldsymbol{u} = 0 \tag{1}$$

$$\boldsymbol{u}|_{\Sigma_{\text{top}}} = \boldsymbol{u}|_{\Sigma_{\text{bottom}}} = 0\ , \quad \omega|_{\Sigma_{\text{inner}}} = \Omega\ , \quad \omega|_{\Sigma_{\text{outer}}} = 0 \tag{2}$$

where $\boldsymbol{u}$, ρ, p and ν are the velocity, the density, the pressure and the kinematic viscosity of the fluid, respectively. Σ_{top}, Σ_{bottom}, Σ_{inner} and Σ_{outer} represent the top end plate, the bottom end plate, the inner and outer cone's walls, respectively, ω is the angular velocity of the cones. We adopt a Cartesian coordinate system $Oxyz$ with z-axis along the axis of rotation and the gravity in the negative z-direction. H, $R_1\,(R_2)$ and α are the hight of device, the radius of the inner (outer) cone at the thickest end and the cone's inclination, respectively.

Parameter definitions: All our numerical results are described in term of following non-dimensional parameters.

Reynolds number: $Re = \mathrm{d}R_1\Omega/\nu$, aspect ratio $\Gamma = H/d$, radius ratio $\eta = R_1/R_2$ and cone's inclination angle α.

2. Outline of the numerical method

2.1 *Outlines*

The nonlinear and time dependent Eqs.(1) together with the boundary conditions (2) and the initial conditions $\boldsymbol{u}|_{t=0} = p|_{t=0} = 0$ are integrated numerically using the finite volumes method. For the convection terms in the equations, a second-order upwind scheme is used to calculate the face values of the various quantities by interpolation from the cell centre values. The central difference quotient is used for the diffusion terms which are always accurate to the second order. The temporal discretization involves integrations of all terms in the differential equations with a time step Δt. The integration of the transient terms is implicit by using a second-order formulation. The SIMPLE algorithm is used to link the pressure and the velocity. The discretized equations are then solved sequentially using a segregated solver. The convergence is achieved when the residual falls below 10^{-4} for the pressure and the three velocity components.

2.2 *Convergence and validation*

The grids used for the numerical simulations consist of tetrahedral elements. Extensive grid-refining tests are conducted by varying the element order and