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Numerical simulation of thermal convection of viscoelastic fluids in an open-top porous medium with constant heat flux*

NIU Jun (牛骏)[1], SHI Zai-hong (石在虹)[1], TAN Wen-chang (谭文长)[2]
1. Petroleum Exploration and Production Research Institute of SINOPEC, Beijing 100083, China,
E-mail: niujun.syky@sinopec.com
2. State Key Laboratory for Turbulence and Complex Systems and Department of Mechanics and Aerospace Engineering, College of Engineering, Peking University, Beijing 100871, China

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Abstract: This paper makes a numerical study of the buoyancy-driven convection of a viscoelastic fluid saturated in an open-top porous square box under the constant heat flux boundary condition. The effects of the relaxation and retardation times on the onset of the oscillatory convection, the convection heat transfer rate and the flow pattern transition are investigated. It is shown that a large relaxation time can destabilize the fluid flow leading to an early onset of the thermal convection and a high heat transfer rate, while a large retardation time tends to stabilize the flow and suppress the convection onset and the heat transfer. After the convection sets in, the flow bifurcation appears earlier with the increase of the relaxation time and the decrease of the retardation time, resulting in more complicated flow patterns in the porous medium. Furthermore, with the increase of the ratio of the relaxation time to the retardation time, the fluid may be blocked from flowing through the open-top boundary, which may be caused by the viscoelastic effect. Finally, the comparison of our results with those under isothermal heating boundary conditions reveals that the heat transfer rate corresponding to a constant heat flux boundary is always higher.

Key words: thermal convection, viscoelastic fluids, porous medium, constant heat flux

Introduction

It is of a great importance to study the thermal convection of viscoelastic fluids in porous media, as its applications are found in many engineering fields, like the paper and textile coating, the composite manufacturing processes, and the bio-engineering[1-4]. In the oil reservoir engineering, the heavy oil is also found to exhibit viscoelastic rheological characteristics[5]. However, the thermal convection of viscoelastic fluids in porous media is far less fruitfully studied than that of the Newtonian fluids[6-8]. The main reason may be due to the lack of a simple model for the description of the viscoelastic flow behavior in porous media as the Darcy's law for the Newtonian fluids.

Recently, the modified Darcy's law[9-12] has drawn much attention in the research of thermal instability problems of viscoelastic fluid saturated porous media heated from below. This modified Darcy's law is a macroscopic phenomenological model, which incorporates the viscoelastic effects suggested by Alisaev and Mirzadjanzade[13] and is based on the rheological behavior of viscoelastic liquids. With the modified Darcy's law, Niu et al.[14] conducted stability analyses of viscoelastic fluids in square porous media under three different heating boundary conditions, which shows that an oscillatory convection mode always sets in before a steady one. Kim et al.[9] conducted linear and non-linear stability analyses of the thermal convection in an Oldroyd-B fluid saturated porous layer with isothermal bottom heating. However, these studies are based on a linear or weakly non-linear analysis, where the Darcy-Rayleigh number should be around the critical values for the onset of the convection.

When the Rayleigh number gets higher and the analytical results become invalid, numerical simulations must be conducted in order to investigate the

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Biography: NIU Jun (1985-), Male, Ph. D., Senior Engineer

flow and heat transfer process. Fu et al.[15] numerically analyzed the thermal convection of a viscoelastic fluid in an impermeable porous square box heated from below with a constant bottom temperature. It is found that after the steady convection sets in, the oscillatory convection tends to be suppressed and the interaction between the two types of thermal convections makes the flow patterns in the porous layer very complicated. Furthermore, the thermal convection of viscoelastic fluids in porous layers would undergo earlier bifurcations with the increase of the Darcy-Rayleigh number than that of the Newtonian fluids and the occurrence of the bifurcation is earlier for a larger ratio of the relaxation time to the retardation time. As a consequence, the heat transfer characteristics are quite different from those of the Newtonian fluids. The heat transfer rate (as represented by the Nusselt number) is essentially higher for a larger ratio of the relaxation time to the retardation time, which obeys different scaling laws delimited by the viscoelastic parameters and the Darcy-Rayleigh number. The thermal convection for a viscoelastic fluid saturated in an open-top (permeable) porous medium with an isothermal heating boundary[16] was also studied, which shows that the onset of the thermal convection occurs earlier than that under a closed-top condition, as the open-top boundary imposes a weaker constraint against the fluid flow inside the porous medium. Therefore, the heat transfer rate after the onset of the thermal convection is larger compared to the closed-top case. However, to the best of our knowledge, no results were obtained for the thermal convection of a viscoelastic fluid in a porous medium under a constant heat flux boundary condition.

To study the thermal convection in viscoelastic fluid saturated open-top porous media under a constant heat flux boundary condition, a numerical simulation was made in our previous letter[17], where the viscoelastic effects on the convection evolution were briefly discussed.

The objective of this paper is to systematically analyze the entire thermal convection process, beginning from the convection onset to the convection bifurcation at high Rayleigh numbers. Therefore, a linear stability analysis is first conducted to study the onset of the thermal convection in a viscoelastic fluid saturated open-top porous media under the constant heat flux boundary condition. Then, a numerical algorithm is built and the corresponding calculation program is developed for analyzing the convection heat transfer and the bifurcation. The influence of different heating conditions is studied by comparing our results with those in the previous work.

1. Model formation

We consider a model of a bounded two-dimensional rectangular porous medium of length a and height H^*, with a permeable isothermal top boundary at temperature T_0^*, where the pressure at the top boundary is assumed as constant[16]. The impermeable bottom boundary is under a constant heat flux q. The two vertical boundaries are adiabatic and impermeable. The porous medium has a permeability K, which is saturated by an incompressible viscoelastic fluid with a constant dynamic viscosity μ, a coefficient of thermal expansion β and a density ρ. The fluid-saturated porous medium has a thermal conductivity k and a thermal diffusivity κ. From the modified Darcy's law, the governing equations of this model are[14-16]

$$\nabla \cdot \boldsymbol{v}^* = 0 \tag{1}$$

$$\left(1+\bar{\lambda}\frac{\partial}{\partial t^*}\right)(-\nabla^* p^* + \rho^* g \boldsymbol{z}) = \frac{\mu}{K}\left(1+\bar{\varepsilon}\frac{\partial}{\partial t^*}\right)\boldsymbol{v}^* \tag{2}$$

$$\frac{\partial T^*}{\partial t^*} + (\boldsymbol{v}^* \cdot \nabla^*)T^* = \kappa \nabla^{*2} T^* \tag{3}$$

$$\rho^* = \rho_0[1-\beta(T^* - T_0)] \tag{4}$$

where $\boldsymbol{v}^* = (u^*, w^*)$ is the Darcy velocity, p^* the pressure, g the gravitational acceleration, $\bar{\varepsilon}$ and $\bar{\lambda}$ are, respectively, the strain retardation time and the stress relaxation time, $\boldsymbol{z}$ is the unit vector along the z-direction which is vertically upward, and ρ_0 the density at temperature T_0. The boundary conditions for this model are

$$\boldsymbol{v}^*(0, z^*) = \boldsymbol{v}^*(a^*, z^*) = \boldsymbol{v}^*(x^*, 0) = 0 \tag{5a}$$

$$p^*(x^*, H^*) = p_0 \tag{5b}$$

$$\frac{\partial T^*(0, z^*)}{\partial x^*} = \frac{\partial T^*(a^*, z^*)}{\partial x^*} = 0 \tag{5c}$$

$$T^*(x^*, H^*) = T_0 \tag{5d}$$

$$-k\frac{\partial T^*(x^*, 0)}{\partial z^*} = q \tag{5e}$$

The constant top pressure boundary condition (5b) is used to describe the open top and is converted into an equivalent velocity boundary condition $(1+\bar{\varepsilon}\partial/\partial t^*)\cdot u^*(x^*, H^*) = 0$ by substituting (5b) into (2).

Our aim is to analyze the whole heat transfer process in the viscoelastic fluid saturated porous medium