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Construction of analytical solutions and numerical methods comparison of the ideal continuous settling model

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ABSTRACT

Continuous sediment has a wide application in chemical engineering process, such as wastewater treatment, water reuse, mineral waste manage and processing. This paper provides analytical solutions of the ideal continuous settling model by using the method of characteristics. The analytical solutions are compared with experiment data to show that the solutions accurately predict the sediment height and concentration as a function of time and loading conditions. Additionally, three alternative methods, using finite differences are compared to the analytical solutions, and their accuracy and efficiency are evaluated. It is shown that all three methods are reliable for solving sedimentation problems but have varying efficiency. Method YRD is the most accurate but also has the greatest computation and implementation cost. Method SG is the least accurate but is the easiest to implement with lowest computation cost. Method G is a compromise between the two methods, providing acceptable accuracy and low computation cost.

1. Introduction

Continuous sedimentation, a gravity driven solid-liquid separation process, has various applications in industrial areas including the wastewater treatment, water reuse, mineral waste manage and processing. However, in current engineering application, the design and operation of the continuous settling tanks still remain as a difficult task, and generally, empirical and conservative strategies are applied, which may cause both capital and land waste, as well as the unanticipated performance flocculation of the settling tank itself (Northcott et al., 2005; Li and Stenstrom, 2014a, 2014c). For the purposes of understanding the continuous settling behavior and optimizing settling tank performance, mathematical models are encouraged to being used, and in most commercial simulators, the ideal one-dimensional (1-D) continuous settling model (without compression effect) is equipped due to its relative well understanding and less computation burden, especially if long term simulation is needed (Bürger et al., 2011).

Given the complexity of real system conditions (*e.g.*, viscosity, dispersion, turbulence, rake effect, various settling behaviors), the concept of the ideal thickener was introduced by Shannon et al. (1963) to simplify the modeling task. In an ideal 1-D condition, the secondary settling tank (SST) possesses a constant cross-section with uniform solids concentration in each horizontal layer, and the complex hydrodynamics are simplified as the upward effluent flow to the top and downward underflow to the bottom, as shown in Fig. 1. The distribution of solids are determined by both gravity settling and the bulk hydraulic transport, and the mass conservation law holding in each layer can be expressed as the partial differential equation, Eq. (1) (Diehl, 1997; Diehl and Jeppsson, 1998):

$$\frac{\partial \phi}{\partial t} + \frac{\partial F(\phi)}{\partial x} = s\delta(x)$$
$$F(\phi) = \begin{cases} -v_e\phi_e = g_e & x < -H \\ f_{bk}(\phi) - v_e\phi = g(\phi) & -H < x < 0 \\ f_{bk}(\phi) + v_u\phi = f(\phi) & 0 < x < D \\ v_u\phi_u = f_u & x > D \end{cases} \tag{1}$$

where F is the flux function, $\delta(z)$ is the Dirac impulse, $\phi(x,t)$ denotes the solid concentration, x is the depth from the feed inlet, t is the time, $s = v_f\phi_f$, denotes the feed solids flux (ϕ_f is the feed solid concentration and v_f is the feed flow velocity), f_{bk} is the Kynch batch flux function and the solid mass fluxes leaving at the effluent weir and bottom are $g_e = v_e\phi_e$ (v_e is the effluent flow velocity and ϕ_e is

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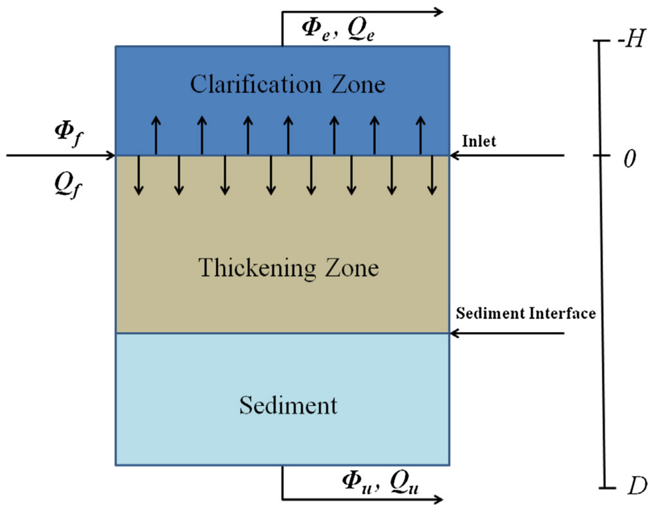

Fig. 1. Schematic overview of ideal continuous settling tank with constant cross-section area

the effluent solids concentration) and $f_u = v_u\phi_u$ (v_u is the underflow velocity and ϕ_u is the underflow solids concentration) respectively.

It is noticeable that Eq. (1) only can be solvable with proper constitutive relations. The fundamental constitutive relation for hindered settling modeling is the Kynch's assumption that the hindered settling velocity is solely determined by the local solids concentration. Based on the Kynch's assumption, three alternative methods have been established to develop the required constitutive function: the hindered settling factor approach (Buscall and White, 1987; Landman et al., 1988; Usher and Scales, 2005; Gladman et al., 2010), the Darcy's Law approach (Karl and Wells, 1999; Kinnear, 2002) and Kynch flux density approach (Bürger et al., 2000, 2005). However, the Kynch's assumption is not a nostrum, since it can only provide a complete settling behavior description of Kynchian suspensions with no compressive behavior at any concentration. Otherwise, its validity can only be proved in hindered settling region, where the concentration is sufficiently low that no weight-bearing network formed (Dixon, 1977).

When in high concentration range, where strong particle-particle interaction exists, compression settling occurs because of the compressive stress transmitted through the formed net structure (de Kretser et al., 2003), and modeling the compression settling process is significant for applications as diverse as thickening, dewatering, filtration and centrifugation. Two parallel theories have been developed to interpret the compression settling: geotechnical approach (Bürger, 2000; Bürger et al., 2001), which quantifies the sediment compressibility by using effective solids stress or the solids pressure; compression rheology approach (Buscall et al., 1987; Buscall and White, 1987), where the compressibility is characterized as the physically measurable network strength: compressive yield stress. The effective solid stress and solid pressure are usually defined as solid volumetric concentration dependent functions rather than the intrinsic material property as the compressive yield stress is. Except for the significant conceptual difference, these two approaches actually have a similar rheological basis, thus making them parallel (de Kretser et al., 2003).

The development of settling theory including the hindered and compression rheology is the first step for model formula complementation, and solving these PDEs, which means accurately solution calculation, is equivalently important for reliable model predications. When hindered settling dominates, the model governing equation can be written as Eq. (1), nonlinear hyperbolic PDEs, known as the convection-dominant model. The compression effect can be modeled by adding a nonlinear diffusion term to Eq. (1), and then the model formula becomes strongly degenerate parabolic PDEs, known as the convection-compression model (Bürger et al., 2012). Though differing in rheology basis, both convection-dominant and convection-compression models possess the similar mathematical characteristics, and solving the compression including model will not greatly increase the solution technique complexity (Bürger et al., 2012). Therefore, from a mathematical point of view, it is informative to fully understand the mathematical implication of Eq. (1) before investigating more complex models (Diehl, 2000).

Based on the mass continuity law and Kynch's assumption, the advantage of Eq. (1) is that it is capable to capture the movement of large concentration discontinuities without knowing their physical mechanisms (Kynch, 1952). However, solution discontinuities, which can be physically interpreted as the concentration gradients, are expected to occur as a function of time and height in solving Eq. (1), and greatly increases the complexity of required solution techniques. Solving Eq. (1) can be either numerical or analytical: numerical techniques including Method G (Jeppsson and Diehl, 1996), Method EO (Bürger et al., 2005), Method YRD (Li and Stenstrom, 2014a, 2014b) et al. have achieved some degree of success in shock capturing and solution calculation, but cannot always satisfy practical application standards, such as high accuracy but low computation burden; the only available approach for analytical solution construction is the method of characteristics (MOC), which avoids complicate discretization procedure but provide high accuracy solutions. Therefore, it is worthwhile further investigating the implementation strategy of MOC in 1-D continuous settling modeling.

The application of MOC to gravity settling problem can trace its history to 1950s, when Kynch (1952) analyzed the solids concentration distribution within the batch settling cylinder by using constant concentration lines, or iso-concentration lines, which is mathematically equivalent to characteristics. Thereafter, this approach was widely applied in practical SST design and operation (Fitch, 1979, 1983, 1993). In recent studies, Diehl (2007) showed that the inverse problem of estimating of the batch settling flux function from experimental data can also be well addressed by using MOC. The first MOC study in continuous settling modeling was provided by Petty (1975) to show that the limiting flux, commonly observed in lab and full scale tests, is an intrinsic nonlinear phenomenon of the governing nonlinear hyperbolic PDEs, which is lately supported by Chancelier et al. (1997) and Diehl (2008), and the propagation of solution discontinuities from bottom boundary is caused by interaction of rarefaction waves. Nevertheless, Petty's work is a MOC based continuous settling behavior analysis more than an analytical solution developing study. Hence, further investigations were motivated to complement the MOC theory in continuous settling study, including the global weak solution construction (Bustos, 1988; Bustos et al., 1990; Diehl, 1997), boundary condition determination (Bustos and Concha, 1992; Diehl, 1996, 2000), and control theory development (Buscall et al., 1982, 1990; Diehl, 2005, 2006).

The first goal of this paper is to construct solutions of the ideal SST model that includes hindered settling and hydraulic bulk transport with dynamic loading conditions on the basis of the previously developed MOC implementation strategy. The MOC solutions are compared with experimental continuous settling data to demonstrate the accuracy of MOC solutions in predicting dynamic continuous settling behaviors. Given that numerical solution techniques are often used for continuous settling models, the second part of this paper focuses on the convergence analysis of three representative numerical methods: Method SG, Method G and Method YRD by using the MOC solutions as reference solutions. Accuracy and computation cost of these three methods are also investigated to compare their efficiency for practical engineering