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A COUPLED MORPHODYNAMIC MODEL FOR APPLICATIONS INVOLVING WETTING AND DRYING*

LIANG Qiuhua
School of Civil Engineering and Geosciences, Newcastle University, Newcastle upon Tyne NE1 7RU, England, UK, E-mail: Qiuhua.Liang@ncl.ac.uk

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Abstract: This work presents a new finite volume Godunov-type model for predicting morphological changes under the rapidly varying flood conditions with wetting and drying. The model solves the coupled shallow water and Exner equations, with the interface fluxes evaluated by an Harten-Lax-van Leer-Contact (HLLC) approximate Riemann solver. Well-balanced solution is achieved using the surface gradient method and wetting and drying are handled by a non-negative reconstruction approach. The new model is validated against several theoretical benchmark tests and promising results are obtained.

Key words: morphodynamic model, Exner equation, shallow water equations, Godunov-type method, wetting and drying, well-balanced scheme

Introduction

Morphodynamics concerns the evolution of the channel bathymetry in response to sediment activities driven by flows. It may have great impacts on local habitats, affect river conveyance and hence have implications for flooding during storm events. An understanding of morphodynamics and the relevant processes is therefore vital for river management systems. In the past few decades, with the rapid development of computer technology, computer modelling has become a common practice in assessing river morphodynamics although it may not entirely replace the laboratory studies for understanding the relevant processes[1].

A morphodynamic modelling system is typically consisted of a hydrodynamic component that describes the flow hydrodynamics and a sediment transport/morphological component that accounts for the bed evolution[2]. Conventionally, morphodynamic modelling is carried out in a decoupled way where the flow hydrodynamics is first obtained and then used to drive sediment transport and update the channel bed change[2-4]. Ignoring the unsteady hydrodynamic effects, this approach has been recognized to be more appropriate for slow-varying flows or quasi-steady processes[5]. For a rapidly-varying flow (e.g., dam breaks, floods, etc.) where the flow unsteadiness can strongly impact the sediment transport, a coupled model, in which the flow hydrodynamics and morphodynamics are solved instantaneously, is desirable and may lead to more reliable predictions[5]. A number of coupled models that solve the integrated shallow water and Exner equations have been reported in recent years for simulating morphodynamics. For example, Liu et al.[6] presented a Roe's approximate Riemann solver based finite volume Godunov-type morphodynamic model on unstructured grids with applications to coastal waters. They also compared the results with those produced by the quasi-steady approach and confirmed the necessity of using a coupled approach for a problem involving strong fluid-sediment interactions. Benkhaldoun et al.[7] reported an alternative finite volume Godunov-type model combined with the Roe's approximate Riemann solver on adaptive triangular grids. The authors also compared the performance of the coupled and decoupled methods and indicated that the coupled approach out-performed the quasi-steady approach when the bed-load rapidly interacts with the hydrodynamics.

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Biography: LIANG Qiu-hua (1974-), Male, Ph. D., Lecturer

Murillo and García-Navarro[8] formulated the sediment fluxes in the Exner equation in a universal form for a number of different empirical formulae and solved the coupled system using a first-order finite volume Roe-type scheme. Soares-Frazão and Zech[9] presented a first-order finite volume Godunov-type morphodynamic model using an Harten-Lax-van Leer-Contact (HLLC) approximate Riemann solver, where new wave-speed estimates were derived based on the novel approximate eigenvalues to the coupled governing equations. Attempts have also been made to solve the coupled system using Essentially Non-Oscillatory (ENO) and Weighted Essentially Non-Oscillatory (WENO) schemes[10], higher-order Central Weighted Essentially Non-Oscillatory (CWENO) scheme[11] and finite element discontinuous Galerkin method[12,13].

However, most of these existing models concerns only wet-bed applications, i.e., the problem domains are covered entirely by water. A morphodynamic model that can handle applications involving wetting and drying may lead to wider applications. Therefore, this work aims to develop a new 2-D shallow flow model to include morphological change under the rapidly varying flood conditions that involves wetting and drying.

1. Governing equations

The coupled governing equations, consisting of the 2-D SWEs and the Exner equation, describing shallow flows over erodible bed may be written in a matrix form as

$$\frac{\partial \boldsymbol{q}}{\partial t}+\frac{\partial \boldsymbol{f}}{\partial x}+\frac{\partial \boldsymbol{g}}{\partial y}=\boldsymbol{s} \tag{1}$$

where t, x and y are the time and the two Cartesian coordinates, $\boldsymbol{q}$ denotes the vector of flow variables, $\boldsymbol{f}$ and $\boldsymbol{g}$ are the flux vectors in the x and y-direction, and $\boldsymbol{s}$ is the vector containing the source terms. The vectors are given by

$$\boldsymbol{q}=\begin{bmatrix} h \\ q_x \\ q_y \\ z \end{bmatrix},\quad \boldsymbol{f}=\begin{bmatrix} q_x \\ \dfrac{q_x^2}{h}+\dfrac{gh^2}{2} \\ \dfrac{q_x q_y}{h} \\ q_{sx} \end{bmatrix},\quad \boldsymbol{g}=\begin{bmatrix} q_y \\ \dfrac{q_x q_y}{h} \\ \dfrac{q_y^2}{h}+\dfrac{gh^2}{2} \\ q_{sy} \end{bmatrix},$$

$$\boldsymbol{s}=\begin{bmatrix} 0 \\ \dfrac{-C_f q_x (q_x^2+q_y^2)}{h^3}-\dfrac{gh\partial z}{\partial x} \\ \dfrac{-C_f q_y (q_x^2+q_y^2)}{h^3}-\dfrac{gh\partial z}{\partial y} \\ 0 \end{bmatrix} \tag{2}$$

where h is the water depth, $q_x=uh$ and $q_y=vh$ are the unit-width discharge in the x and y-direction with u and v being the corresponding depth-averaged velocities, z represents the bed elevation above an arbitrary datum, $g=9.81\,\mathrm{m/s^2}$ denotes the acceleration due to gravity, $q_{sx}=\zeta A_g q_x \bullet \left(q_x^2+q_y^2\right)/h^3$ and $q_{sy}=\zeta A_g q_y\left(q_x^2+q_y^2\right)/h^3$ are the sediment fluxes, $\zeta=1/(1-p)$ is a fixed parameter with p being the bed porosity, A_g $(0<A_g\le 1)$ is an empirical constant that indicates a stronger flow-sediment interaction as it reaches 1, $C_f=gn^2/h^{1/3}$ is the bed roughness coefficient with n being the Manning coefficient, $-\partial z/\partial x$ and $-\partial z/\partial y$ are the two bed slopes.

In order to construct a proper eigenstructure, the coupled System (1) and (2) are rewritten as follows by moving the bed slope source terms to the left hand side[9]

$$\frac{\partial \boldsymbol{q}}{\partial t}+\frac{\partial \boldsymbol{f}(\boldsymbol{q})}{\partial x}+\boldsymbol{H}(\boldsymbol{q})\frac{\partial \boldsymbol{q}}{\partial x}+\frac{\partial \boldsymbol{g}(\boldsymbol{q})}{\partial y}+\boldsymbol{K}(\boldsymbol{q})\frac{\partial \boldsymbol{q}}{\partial y}=\boldsymbol{s}_f \tag{3}$$

where

$$\boldsymbol{H}(\boldsymbol{q})=\begin{bmatrix} 0&0&0&0\\ 0&0&0&c^2\\ 0&0&0&0\\ 0&0&0&0 \end{bmatrix},\quad \boldsymbol{K}(\boldsymbol{q})=\begin{bmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&0&c^2\\ 0&0&0&0 \end{bmatrix},$$

$$\boldsymbol{s}_f=\begin{bmatrix} 0 \\ -C_f q_x \dfrac{\left(q_x^2+q_y^2\right)}{h^3} \\ -C_f q_y \dfrac{\left(q_x^2+q_y^2\right)}{h^3} \\ 0 \end{bmatrix} \tag{4}$$

where $c=\sqrt{gh}$. Then the coupled system may be written in a quasi-linear form as

$$\frac{\partial \boldsymbol{q}}{\partial t}+\tilde{\boldsymbol{A}}(\boldsymbol{q})\frac{\partial \boldsymbol{q}}{\partial x}+\tilde{\boldsymbol{B}}(\boldsymbol{q})\frac{\partial \boldsymbol{q}}{\partial y}=\boldsymbol{s}_f \tag{5}$$

where $\tilde{\boldsymbol{A}}(\boldsymbol{q})$ and $\tilde{\boldsymbol{B}}(\boldsymbol{q})$ are the pseudo-Jacobian matrices given by[9]

$$\tilde{\boldsymbol{A}}(\boldsymbol{q})=\frac{\partial \boldsymbol{f}(\boldsymbol{q})}{\partial \boldsymbol{q}}+\boldsymbol{H}(\boldsymbol{q})=\begin{pmatrix} 0&1&0&0\\ c^2-u^2&2u&0&c^2\\ -uv&v&u&0\\ M&N&R&0 \end{pmatrix},$$