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Improving scenario decomposition algorithms for robust nonlinear model predictive control

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ABSTRACT

This paper deals with the efficient computation of solutions of robust nonlinear model predictive control problems that are formulated using multi-stage stochastic programming via the generation of a scenario tree. Such a formulation makes it possible to consider explicitly the concept of recourse, which is inherent to any receding horizon approach, but it results in large-scale optimization problems. One possibility to solve these problems in an efficient manner is to decompose the large-scale optimization problem into several subproblems that are iteratively modified and repeatedly solved until a solution to the original problem is achieved. In this paper we review the most common methods used for such decomposition and apply them to solve robust nonlinear model predictive control problems in a distributed fashion. We also propose a novel method to reduce the number of iterations of the coordination algorithm needed for the decomposition methods to converge. The performance of the different approaches is evaluated in extensive simulation studies of two nonlinear case studies.

1. Introduction

The use of optimization for the control of chemical processes is a standard technique in the process industry because it makes it possible to take into account various process objectives, such as economic ones, and calculate the best control actions subject to constraints that arise from quality, safety or environmental requirements (Engell, 2007). A family of the methods termed model predictive control (MPC) approaches evolved in this respect.

In recent years, a significant amount of research has been devoted to the use of economic cost functions within the framework of nonlinear model predictive control (NMPC), see e.g. Rawlings and Amrit (2009), Idris and Engell (2012), Prada et al. (2008), Gopalakrishnan and Biegler (2013). The optimal operation of a system according to an economic cost corresponds usually to driving the system to its constraints. For this reason, plant model mismatch or disturbances (which are always present in reality) can lead easily to constraint violations and the explicit consideration of uncertainty in the design of the MPC controller becomes very important.

The first efforts in robust MPC tried to address this problem focusing on the so-called min-max MPC (Campo and Morari, 1987). This approach obtains a sequence of control inputs that minimizes the cost of the worst-case realization of the uncertainty while satisfying the constraints for all the cases of the uncertainty. Min-max MPC however does not take into account the fact that new information will be available in the future and therefore the result may be overly conservative and may lead to infeasible optimization problems, as illustrated in Scokaert and Mayne (1998). Different methods such as the closed-loop (or feedback) min-max NMPC in Lee and Yu (1997) and Mayne (2001), or tube-based MPC (Mayne et al., 2005; Rawlings and Amrit, 2009; Rakovic et al., 2011) have been proposed to overcome the limitations of open-loop min-max MPC. However, most of the methods above cannot be applied to realistic problems, because they result in prohibitive computational cost, or they cannot be easily designed for nonlinear systems or because they are very conservative.

A different possibility is to consider the integration of the stochastic programming paradigm (Birge, 1997; Shapiro, 2009) into the framework of model predictive control. This idea has been used for the linear MPC case in Scokaert and Mayne (1998),

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Muñoz de la Peña et al. (2005), Bernardini and Bemporad (2009) as well as to NMPC (Lucia et al., 2013a), denoted as multi-stage NMPC which is a robust NMPC approach based on the assumption that the uncertainty can be modeled by a scenario tree. The application of multi-stage NMPC has recently provided very promising results and it is the approach to robust NMPC followed in this work.

The main drawback of the approach is that the size of the resulting optimization problem grows exponentially with the length of the prediction horizon, and with the number of uncertainties as well as with the number of different values of each uncertainty that is considered in the design of the scenario tree. For this reason, an efficient solution of the resulting Nonlinear Programming (NLP) problem is necessary to make it possible to solve such problems in real time. The main goal of this paper is to analyze the different possibilities to solve the multi-stage NMPC in an efficient way, both in terms of computation time and memory requirements.

For this purpose, we take advantage of the fact that each scenario in the scenario tree is an independent subproblem except for the non-anticipativity (or causality) constraints which make sure that the optimal inputs do not anticipate the realization of the uncertainty in the present sampling instant since it is unknown. We investigate different possibilities to relax these constraints and to solve the individual optimization problems within a coordination algorithm that enforces satisfaction of the non-anticipativity constraints upon its convergence.

We present classical coordination algorithms in the framework of economic multi-stage NMPC and propose a novel distributed algorithm that uses sensitivity information to reduce the number of iterations of the coordination algorithm needed to converge. The different distributed multi-stage NMPC algorithms are analyzed via extensive simulation studies of two industrial case studies, extending the results presented in Marti et al. (2015).

The remainder of the paper is organized as follows. Section 2 summarizes the concept of multi-stage NMPC. The different coordination algorithms used in this work are presented in Section 3 and a possible modification to achieve convergence in less iterations of the coordination algorithms is presented in Section 4. The resulting distributed NMPC algorithms are evaluated in Section 5 using an industrial hydrodesulphurization example and in Section 6 using an industrial polymerization reactor. The paper is concluded in Section 7.

2. Multi-stage NMPC

This section reviews the main concepts of the multi-stage NMPC approach presented in Lucia et al. (2013a, 2014b).

In multi-stage NMPC, the model uncertainty is taken into account by considering a tree of discrete scenarios for each possible value of the uncertainty as depicted in Fig. 1. The formulation of a scenario tree makes it possible to take explicitly into account that the future decisions can depend on the new information (measurements) that will become available in the future. Thus the future control inputs can be adapted according to the future realizations of the uncertainty and the conservativeness of the approach is reduced compared to other robust methods that search for a single sequence of control inputs to satisfy the constraints for all the possible values of the uncertainty. Formulating the uncertain decision process as a scenario tree is a well-known approach in the field of multi-stage stochastic programming, which has been extensively used in decision theory and finances (Shapiro, 2009). In the case that the uncertainty is truly discrete-valued, this is the best solution possible for a given prediction horizon. Generally this is not the case, and multi-stage NMPC is an approximation of the best solution.

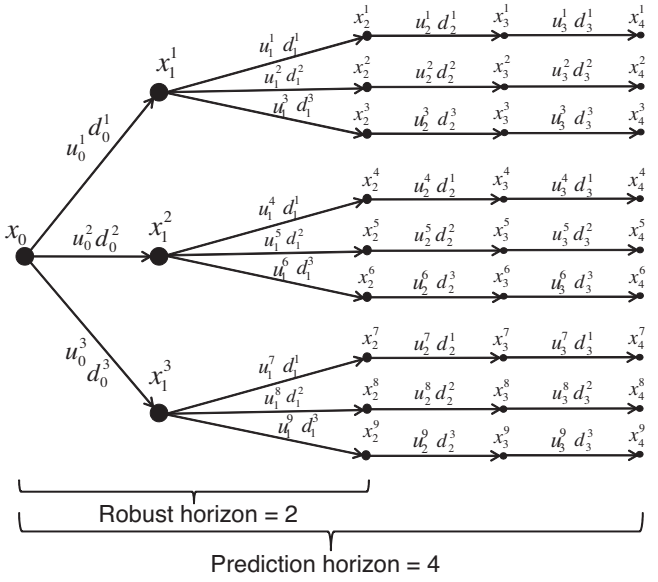

Fig. 1. Scenario tree representation of the uncertainty evolution for multi-stage NMPC.

To formulate mathematically the multi-stage NMPC approach, we consider a discrete-time nonlinear system:

$$\boldsymbol{x}_{k+1}^{j} = \boldsymbol{f}(\boldsymbol{x}_{k}^{p(j)}, \boldsymbol{u}_{k}^{j}, \boldsymbol{d}_{k}^{r(j)}), \tag{1a}$$

where each state vector $\boldsymbol{x}_{k+1}^{j} \in \mathbb{R}^{n_x}$ at stage $k+1$ and position j depends on the parent state (node) $\boldsymbol{x}_{k}^{p(j)}$ at stage k, the vector of control inputs $\boldsymbol{u}_{k}^{j} \in \mathbb{R}^{n_u}$ and the corresponding realization r of the uncertainty $\boldsymbol{d}_{k}^{r(j)} \in \mathbb{R}^{n_d}$ (e.g. in Fig. 1, $\boldsymbol{x}_{2}^{6} = \boldsymbol{f}(\boldsymbol{x}_{1}^{2}, \boldsymbol{u}_{1}^{6}, \boldsymbol{d}_{1}^{3})$). The uncertainty at the stage k is defined by $\boldsymbol{d}_{k}^{r(j)} \in \{\boldsymbol{d}_{k}^{1}, \boldsymbol{d}_{k}^{2}, \ldots, \boldsymbol{d}_{k}^{s}\}$ for s different possible combinations of values of the uncertainty. We define the set of indices (j, k) in the scenario tree as $\boldsymbol{I}$. $\boldsymbol{S}_i$ denotes the ith scenario defined as the path from the root node $\boldsymbol{x}_0$ to one of the leaf nodes and it contains all the states $\boldsymbol{x}_{k}^{j}$ and control inputs $\boldsymbol{u}_{k}^{j}$ that belong to the ith scenario.

A common way to build a scenario tree is to consider, as possible branches, a combination of values among the extreme and nominal values of all the uncertainties. For the general nonlinear case, it is not guaranteed that this results in robust constraint satisfaction for the values of the uncertainty that are not considered in the tree, but it has been shown to give very good results in practice (Lucia et al., 2012, 2013a, 2014a,b). If a rigorous guarantee for robust constraint satisfaction of all the possible values of the uncertainty (including those that are not in the tree) is required, the multi-stage approach can be combined with reachability analysis as shown in Lucia et al. (2014c).

Generating the scenario tree in a systematic way (considering the extrema of the uncertainty space) makes the size of the resulting optimization problem to grow rapidly with increasing length of prediction horizon N_p and with increasing number of uncertainties with resulting number of scenarios $N = s^{N_p n_d}$. A possible strategy to avoid the exponential growth of the scenario tree over the prediction horizon is to consider that the uncertainty remains constant after a certain stage (called robust horizon N_r) until the end of prediction horizon (Fig. 1) which gives $N = s^{N_r n_d}$.

The optimization problem that has to be solved at each sampling instant can be written as:

$$\min_{\boldsymbol{x}_{k+1}^{j}, \boldsymbol{u}_{k}^{j}, \ \forall (j,k) \in \boldsymbol{I}} \ \sum_{i=1}^{N} \omega_i J_i(\boldsymbol{X}_i, \boldsymbol{U}_i) \tag{2a}$$