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Derivative-free methods applied to daily production optimization of gas-lifted oil fields

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ABSTRACT

In the oil industry, computer simulators are routinely applied on day to day operations and in what-if analyses. With the increasing complexity of operations, engineers are relying on simulation to synthesize models for mathematical optimization or else applying derivative-free methods for simulation-based optimization, which requires only the sampling of the objective function. This work adapts an algorithm based on augmented Lagrangian and derivative-free trust-region algorithms to handle hard constraints found in production optimization. The effectiveness of the proposed method is assessed in an oil production network and compared with mathematical optimization based on piecewise-linear models.

1. Introduction

The production optimization of oil fields entails solving challenging problems that have been receiving attention of researchers and practitioners. A common practice for production maximization consists in developing models of wells, risers, separators and other equipment which are then used in mathematical optimization (Alarcón et al., 2002; Silva and Camponogara, 2014). The identification and validation of such mathematical models are rather complex, relying on simulation analysis and field measurements. As numerical simulators are intensively used in day-to-day analyses, they are periodically tunned to ensure accuracy and reproduce the observed field behavior. One modeling approach uses the simulator to build piecewise-linear models that yield predictions with the desired accuracy. With such models, production optimization is approximated with a Mixed-Integer Linear Programming (MILP) problem to which specialized and off-the-shelf algorithms can be readily applied (Codas et al., 2012). However, piecewise-linear models can be excessively large to ensure the desired accuracy, particularly for multidimensional functions. Also, the design of effective piecewise-linear approximations depends on strong knowledge of both the Physics and advanced modeling strategies (Vielma et al., 2010). The difficulties involved in obtaining mathematical models make attractive the optimization procedures that use simulators directly, what is known as simulation-based optimization (Sandu and Zhang, 2008; Gunnerud et al., 2013). Gradient-based optimization can be used if the gradients are provided by the simulators or accurately approximated from outside. Derivative-free methods, on the other hand, rely solely on function values calculated by simulation, without the need of gradients.

Traditionally, derivative-free methods are a part of the unconstrained optimization literature, but can be extended to handle certain classes of constraints. Particularly, Kolda et al. (2006) developed a direct-search method using augmented Lagrangian (Conn et al., 1996) to handle nonlinear constraints. This work follows a similar strategy, but the augmented Lagrangian subproblem is solved with the trust-region method of Conn et al. (2009), which was adapted in this work to deal with linear constraints.

Derivative-free trust-region methods work by sampling the objective function within a neighborhood of the incumbent solution. The model of the objective obtained from the samples is valid locally, inside the so-called trust-region. Then, an algorithm can produce the next trial solution by optimizing over the trust-region. These methods differ depending on the shape of the trust-region, the kind of model for the objective, and model maintenance.

In this work, a derivative-free algorithm was applied to maximize oil production of an offshore gas-lifted oil field, which was modeled with a multiphase-flow simulator widely employed in the petroleum industry. The production platform operates several wells with subsea completion, being subject to physical limits which were modeled as nonlinear constraints, such as bounds on

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flow processing and compression capacity. To assess the merit of the derivative-free methods, experiments were performed by varying the operating conditions of the field. Further, an MILP program approximating the production system was obtained by piecewise-linearizing the various models and then solved with an optimization solver. A comparison between MILP and derivative-free optimization indicates that the latter can be effective.

2. Problem definition

This work focuses on the problem of maximizing oil production of offshore oil fields operated by gas-lift, while accounting for physical and operational constraints. Gas-lift consists in injecting high pressure gas at the bottom of production wells to increase production. The oil field encompasses *N* gas-lifted wells, each connected to one of *M* manifolds. The production flow of each manifold is routed to a dedicated separator and split in oil, gas, and water streams. The gas produced is compressed to be re-injected or exported.

Let q_{inj}^n be the flow of gas injected into well n and $q_{\text{inj}} = (q_{\text{inj}}^1, \ldots, q_{\text{inj}}^N)$ be the vector with the injection rates for all of the wells. Because the wells that are connected to a given manifold interact, the actual production of each well is a function of the vector q_{inj}. The total oil and gas gathered by a manifold m are denoted by $q_o^{\text{manif},\,m}\left(q_{\text{inj}}\right)$ and $q_g^{\text{manif},\,m}\left(q_{\text{inj}}\right)$, respectively. The problem of optimizing the oil production subject to constraints on gas compression and lift-gas availability is formulated as follows:

$$\max f = \sum\nolimits_{m=1}^{M} q_o^{\text{manif},\,m}\left(q_{\text{inj}}\right) \tag{1a}$$

$$\text{s.t.} \sum_{n=1}^{N} q_{\text{inj}}^n \le q_{\text{inj}}^{\max} \tag{1b}$$

$$l_n \le q_{\text{inj}}^n \le u_n, \quad n = 1, \ldots, N \tag{1c}$$

$$\sum\nolimits_{m=1}^{M} q_g^{\text{manif},\,m}\left(q_{\text{inj}}\right) \le q_g^{\text{comp}}, \tag{1d}$$

where:

- The values l_n and u_n are the lower and upper bounds of gas injection for well n, while $q_{\text{inj}}^{\max}$ is the total gas available for injection in the field.
- The gas compression capacity of the field, denoted q_g^{comp}, establishes an upper bound on the total gas production.

Additionally, the total gas injected and total gas produced are defined as

$$q_{\text{inj}}^{\text{total}} = \sum\nolimits_{n=1}^{N} q_{\text{inj}}^n, \quad q_g^{\text{total}} = \sum\nolimits_{m=1}^{M} q_g^{\text{manif},\,m}\left(q_{\text{inj}}\right).$$

The difference between these two quantities is the remaining gas, which is not re-injected but exported from the platform:

$$q_g^{\text{export}} = q_g^{\text{total}} - q_{\text{inj}}^{\text{total}}.$$

In this work, all the *N* wells are in production. The production functions of gas and oil gathered by the manifolds, respectively $q_g^{\text{manif},\,m}$ and $q_o^{\text{manif},\,m}$, are obtained directly from simulation. That is, given a vector of lift-gas flows q_{inj}, the simulator computes the production of each well (consequently, the production in each manifold) considering their productivity and the interactions, which occur through shared manifolds and pressure drops in pipelines.

The production optimization problem addressed in this work is representative of real-world offshore oil fields. Also, the derivative-free optimization approach is not limited to this particular problem, allowing other physical constraints to be directly enforced by the simulator or explicitly imposed by algebraic constraints. For instance, constraints can be introduced to restrict any variable that is available by simulation, such as pressures and flow rates.

3. Derivative-free algorithms

Notice that, without the nonlinear constraint (1d) which imposes a limit on compression capacity, the feasible region of the production optimization problem (1) would consist of a polyhedron on the decision variables q_{inj}. An additional difficulty with respect to constraint (1d) is the fact that the left-hand side, which corresponds to q_g^{total}, is not known explicitly, but rather calculated by simulation. Although there exist derivative-free methods which handle such "black-box" constraints (Abramson et al., 2009), our approach relies on the augmented Lagrangian for general constraints, in which nonlinear constraints are substituted by penalizations in the objective function. The resulting problems are then solved by a derivative-free trust-region method that handles linear constraints on the variables.

The augmented Lagrangian algorithm chosen is based on (Conn et al., 1996), adapted to address problems of the form

$$\min f(x) \tag{2a}$$

$$\text{s.t. } Ax - b \le 0 \tag{2b}$$

$$c(x) \le 0 \tag{2c}$$

where $f : \mathbb{R}^n \to \mathbb{R}$, $c : \mathbb{R}^n \to \mathbb{R}^m$, $A \in \mathbb{R}^{p \times n}$, and $b \in \mathbb{R}^p$.

In place of solving this problem directly, the nonlinear constraints $c_1, \ldots, c_m$ are partitioned into q disjoint subsets $\{\mathcal{Q}_j\}_{j=1}^q$ and included as penalties in the objective, yielding the *augmented Lagrangian function*[1]

$$\mathcal{L}(x, \lambda, \mu) = f(x) + \sum_{j=1}^{q} \frac{\mu_j}{2} \sum_{i \in \mathcal{Q}_j} \left[\max\left(0, \lambda_i + \frac{1}{\mu_j} c_i(x)\right)^2 - \lambda_i^2 \right]. \tag{3}$$

where $\lambda = (\lambda_1, \ldots, \lambda_m)$ are Lagrange multiplier estimates and the $\mu_j > 0$ is the parameter that penalizes the violation of constraints in set $\mathcal{Q}_j$.

The method consists of solving a sequence of problems of the form

$$\min_x \mathcal{L}(x, \lambda, \mu) \tag{4a}$$

$$\text{s.t. } Ax - b \le 0, \tag{4b}$$

where λ and μ are kept constant. The solution to this problem is an approximation to the solution of the original problem, provided that either the penalty μ is low (infeasibility is highly penalized) or the Lagrange multiplier estimates λ are sufficiently accurate with regards to those of a stationary point—one satisfying KKT conditions. If the solution to subproblem (4) does not satisfactorily solve the original problem (2), the Lagrange multipliers λ must be improved or the penalty parameter μ has to be lowered. Then, problem (4) is solved again with the updated parameters. This method can be summarized by the following steps:

- Solve the augmented Lagrangian subproblem (4)
- In case the solution is not satisfactory:
  - Update Lagrange multiplier estimates
  - Update penalty parameters

[1] This augmented Lagrangian differs from the original of (Conn et al., 1996) so as to handle inequalities without explicit use of slack variables, as discussed by Bertsekas (1999).