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A SEMI-CIRCLE THEOREM FOR UNSTABLE PERTURBATION IN TROPICAL CYCLONE-SCALE VORTICLES*

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Abstract: In this paper, we discusses the spectrum distribution of baroclinic disturbances in tropical cyclone-scale vortices with the assumption that the basic flow does not vary with height. The semi-circle theorem for the unstable perturbation is obtained and the upper bound of growth rate can be estimated. The upper bound of instability growth rate increases with the increase of the maximum basic-state tangential wind speed, angular speed, inertial parameter, absolute vorticity, radial gradient of absolute vorticity and static stability. The upper bound of instability growth rate is greater for vortices of larger horizontal scale and smaller vertical scale and lower wave-number disturbances.

Key words: tropical cyclone-scale vortex, spiral bands, instability, semi-circle theorem

1. Introduction

Tropical Cyclone (TC) is commonly regarded as an asymmetric vortex, and the spiral bands in it, which are very evident in radar images and satellite images, among the most important asymmetric features of the TC[1]. In fact, a severe weather, such as rainstorm, thunderstorm, and gust, etc., is often accompanied with spiral bands, so many meteorologists pay a great attention to them[2,3]. Instability in a tropical cyclone-scale vortex is an important mechanism for the formation of spiral bands[4,5]. Case studies of specific basic flows, solved by analytical or numerical methods, provide a range of wavenumber in which instability occurs and show the dependence of growth rate on wavenumber and environmental parameters. A more general approach has to provide theoretical limits to growth rates of instabilities. In one line of this approach, the so-called semicircle bounds are obtained for the complex phase speeds of unstable disturbances. Such bounds are of important practical interest because they place some limits on the growth of tropical cyclone-scale vortices.

Howard established a remarkable theorem about plane-wave instabilities in a stratified parallel shear flow[6], which states that the complex phase speed of an unstable disturbance lies within a semicircle on the complex plane centered on a speed halfway between an upper and a lower bounds of the range of velocities that spans the shear flow, and with a radius equal to half the difference between these bounds. Pedlosky extended this theorem to cover quasi-geostrophic baroclinic disturbances in rotating, stratified

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geophysical flows on a β plane[7,8]. The effect of the earth's curvature amounts to extending the radius of instability of the semicircle by a value depending on the phase speed of a barotropic Rossby wave of the wavenumber under consideration. In the limit of very long waves (low wavenumber), the barotropic Rossby wave phase speed is unbounded so that the extended semicircle radius is likewise unbounded. This limitation on the complex phase speed (and growth rate) of a baroclinic instability is not very useful. Cavallini et al. obtained a bounding semicircle for the complex phase speeds of long-wave disturbances in zonal flows on the sphere (or β plane), with a radius twice that of the Howard semicircle and with a center in the minimum mean flow[9].

Meso-scale atmosphere is ageostrophic and includes not only vortex waves, but also inertia-gravitational waves. Even if the beta-effect is ignored, vortex waves still exist as long as the basic flow has horizontal or vertical shear[10]. Therefore, Zhang et al. established a semi-circle theorem about the instability of baroclinic two-dimensional meso-scale perturbations, provided that the basic flow is merely a linear function of height z [11].

However, in the tropical cyclone-scale vortices, the basic flow has not only vertical shear, but also horizontal shear, so the problem becomes more complicated. Shapiro and Montgomery[12] introduced an Asymmetric Balance (AB) model to describe rapidly rotating baroclinic vortices. A nice feature of the AB model is that it incorporates the full curvature effect, which is absent in both quasi-geostropic and semi-geostrophic models. Montgomery and Shaprio[13] used the AB model to investigate the stability of rapidly rotating baroclinic vortices. Their stability analyses can explain many important dynamic mechanisms of hurricane. However, such analyses assume that disturbances are of the form of regular normal modes. Ren[14] extended these stability results to general disturbances and derived a semicircle theorem for bounds of the angular phase speed and growth rate in unstable normal modes.

In order to unveil more general instability properties of tropical cyclone-scale vortices, we use an asymmetric hurricane equations of Hodyss and Nolan[15] in the absence of dissipation and diabatic heating. We consider only the case when the basic flow is merely a function of radius. In Section 2, the model and simplification method will be introduced. Section 3 describes our method for deriving the semicircle theorem. The upper bound of growth rate is estimated in Section 4. Conclusions are presented in Section 5.

2. The model

Our starting point is the radially modified form of the anelastic momentum equations in cylindrical coordinates:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \lambda}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}-fv= -\frac{1}{\bar{\rho}}\frac{\partial p}{\partial r} \tag{1a}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \lambda}+w\frac{\partial v}{\partial z}+\frac{uv}{r}+fu= -\frac{1}{\bar{\rho}r}\frac{\partial p}{\partial \lambda} \tag{1b}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial w}{\partial \lambda}+w\frac{\partial w}{\partial z}= -\frac{1}{\bar{\rho}}\frac{\partial p}{\partial z}+B' \tag{1c}$$

$$\frac{1}{r\bar{\rho}}\frac{\partial (r\bar{\rho}u)}{\partial r}+\frac{1}{r}\frac{\partial v}{\partial \lambda}+\frac{1}{\bar{\rho}}\frac{\partial (\bar{\rho}w)}{\partial z}=0 \tag{1d}$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial r}+\frac{v}{r}\frac{\partial \theta}{\partial \lambda}+w\frac{\partial \theta}{\partial z}=0 \tag{1e}$$

Here radial, azimuthal, and vertical coordinates are denoted by r, λ, z, u, v, w have their usual meaning, p is the pressure, f denotes a constant Coriolis parameter (e.g., $0.5\times10^{-4}\text{s}^{-1}$), $\bar{\rho}$ is the anelastic average density field, $B'=g(\theta-\bar{\theta})/\bar{\theta}$ is the buoyancy term, where θ is the potential temperature, and $\bar{\theta}$ is the basic-state potential temperature.

For simplicity, only unstable perturbations in the vortex with the basic flow not varying with the height will be considered in this paper, so we may set:

$$w(r,\lambda,z,t)=w'(r,\lambda,z,t) \tag{2a}$$

$$v(r,\lambda,z,t)=\bar{v}(r)+v'(r,\lambda,z,t) \tag{2b}$$

$$w(r,\lambda,z,t)=w'(r,\lambda,z,t) \tag{2c}$$