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APPLICATION OF A MODIFIED QUICK SCHEME TO DEPTH-AVERAGED k-ε TURBULENCE MODEL BASED ON UNSTRUCTURED GRIDS*

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Abstract: The modified QUICK scheme on unstructured grid was used to improve the advection flux approximation, and the depth-averaged $k-\varepsilon$ turbulence model with the scheme based on FVM by SIMPLE series algorithm was established and applied to spur-dike flow computation. In this model, the over-relaxed approach was adopted to estimate the diffusion flux in view of its advantages in reducing errors and sustaining numerical stability usually encountered in non-orthogonal meshes. Two spur-dike cases with different defection angles (90^{o}and 135^{o}) were analyzed to validate the model. Computed results show that the predicted velocities and recirculation lengths are in good agreement with the observed data. Moreover, the computations on structured and unstructured grids were compared in terms of the approximately equivalent grid numbers. It can be concluded that the precision with unstructured grids is higher than that with structured grids in spite that the CPU time required is slightly more with unstructured grids. Thus, it is significant to apply the method to numerical simulation of practical hydraulic engineering.

Key words: unstructured grid, modified QUICK, FVM, $k-\varepsilon$ turbulence model, spur-dike

1. Introduction

The unstructured grid has become more prevalent in the computation of hydraulic engineering due to its adapting to complex domain boundaries. It is also allowed for the use of mixed grids consisting of cells with various shapes in unstructured grid without any redundant complication in formulation based on the FVM. However, some high-order schemes on uniform grid are not easy to apply directly to unstructured grid situations. Thus, it is attractive that the high-order schemes can be further applied on unstructured grid to improve computational precision for practical hydraulic situations.

The QUICK is a third-order approximation of the convection term. For unstructured grids, searching exact locations of the upstream nodes will increase the complexity of geometrical data structure and require more computational memory and CPU time. Davidson[1] introduced a modified QUCIK scheme. The upstream nodes for the QUICK are generated from intersections from the line of two adjacent central points and its corresponding interface. In diffusive flux calculation, the normal derivation can not be obtained directly for the mesh skewness. Thus, some methods have been developed to solve the problem such as the directional gradient method[2], Green's theorem method [3-5], imitating momentum interpolation method [6,7], normal derivative method [6,8] simplified normal derivative method [9], minimum correction approach [6,10] and over-relaxed approach [6 ,9,11-13]. In this work, the over-relaxed approach is adopted for the approximation of the cross-diffusion as well as in the pressure-correction equation derived

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from the continuity constraint along with Rhie-Chow momentum interpolation method [14]. The generalized minimum residual (GMRES) [15] method with the incomplete LU (ILUT) precondition is used to accelerate the convergence of solving the linear equations.

Based on unstructured meshes, the above-mentioned method is adopted to discretize the depth-averaged $k-\varepsilon$ turbulence model based on the FVM with the SIMPLE series algorithm. The computation for this model is carried out for two different defection angles of spur-dike and validated by their corresponding experimental data. Additionally, the computational precision and CPU time on structured and unstructured grids are compared.

2. Numerical methods

2.1 *Governing equations*

The continuity equation:

$$\frac{\partial(\rho H)}{\partial t}+\frac{\partial(\rho Hu)}{\partial x}+\frac{\partial(\rho Hv)}{\partial y}=0 \tag{1}$$

The momentum equations:

$$\frac{\partial(\rho Hu)}{\partial t}+\frac{\partial(\rho Huu)}{\partial x}+\frac{\partial(\rho Huv)}{\partial y}= -\rho gH\frac{\partial h}{\partial x}+\frac{\partial}{\partial x}\left(H\mu_e\frac{\partial u}{\partial x}\right)+ \frac{\partial}{\partial y}\left(H\mu_e\frac{\partial u}{\partial y}\right)+\tau_{sx}-\tau_{bx} \tag{2}$$

$$\frac{\partial(\rho Hv)}{\partial t}+\frac{\partial(\rho Hvu)}{\partial x}+\frac{\partial(\rho Hvv)}{\partial y}= -\rho gH\frac{\partial h}{\partial x}+\frac{\partial}{\partial x}\left(H\mu_e\frac{\partial v}{\partial x}\right)+ \frac{\partial}{\partial y}\left(H\mu_e\frac{\partial v}{\partial y}\right)+\tau_{sy}-\tau_{by} \tag{3}$$

The turbulence kinetic energy equation:

$$\frac{\partial(\rho Hk)}{\partial t}+\frac{\partial(\rho Huk)}{\partial x}+\frac{\partial(\rho Hvk)}{\partial y}= \frac{\partial}{\partial x}\left(\frac{\mu_e}{\sigma_k}H\frac{\partial k}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\mu_e}{\sigma_k}H\frac{\partial k}{\partial y}\right)+ H\left(P_K+P_{KV}-\rho\varepsilon\right) \tag{4}$$

The turbulence dissipation rate equation:

$$\frac{\partial(\rho H\varepsilon)}{\partial t}+\frac{\partial(\rho Hu\varepsilon)}{\partial x}+\frac{\partial(\rho Hv\varepsilon)}{\partial y}= \frac{\partial}{\partial x}\left(\frac{\mu_e}{\sigma_\varepsilon}H\frac{\partial \varepsilon}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\mu_e}{\sigma_\varepsilon}H\frac{\partial \varepsilon}{\partial y}\right)+ H\left(C_{1\varepsilon}\frac{\varepsilon}{k}P_K+P_{\varepsilon V}-C_{2\varepsilon}\rho\frac{\varepsilon^2}{k}\right) \tag{5}$$

where u and v are the depth averaged velocity components in the x and y directions respectively, ρ is the water density, h is the free surface water level and H is the full depth, τ_{sx} and τ_{sy} are the surface wind shear stresses along the x and y directions, τ_{bx} and τ_{by} are the bottom bed shear stresses, k and ε denote the turbulence kinetic energy and turbulence kinetic dissipation rate. The constants $C_{\varepsilon 1}, C_{\varepsilon 2}, \sigma_k, \sigma_\varepsilon$ in Eqs.(4) and (5) are set as 1.44, 1.92, 1.0, 1.32 respectively, and μ_e is the effective coefficient of eddy-viscosity which is set as $\mu_e=\mu+\rho C_\mu k^2/\varepsilon$, $C_\mu=0.09$ and μ is the molecular viscosity, P_K is the turbulence production term and is written as

$$P_K=\frac{\mu_e}{H^2}\left[2\left(\frac{\partial Hu}{\partial x}\right)^2+2\left(\frac{\partial Hv}{\partial x}\right)^2+\left(\frac{\partial Hu}{\partial y}+\frac{\partial Hv}{\partial x}\right)^2\right] \tag{6}$$

P_{KV} and $P_{\varepsilon V}$ are new sources taken by the integral of average along the depth direction and are respectively set as

$$P_{KV}=\rho g n^2 H^{-4/3}(u^2+v^2)^{3/2} \tag{7}$$

$$P_{\varepsilon V}=3.6C_{2\varepsilon}C_\mu^{0.5}\rho(u^2+v^2)^2(gn_0^2)^{5/4}H^{-29/12} \tag{8}$$

where n_0 is the coefficient of roughness.