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NUMERICAL SIMULATION OF THE OSCILLATORY FLOW AROUND TWO CYLINDERS IN TANDEM

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ABSTRACT: A numerical study on oscillatory flow past two cylinders in tandem is carried out. The Reynolds-averaged Navier-Stokes equations are solved using a finite element method (FEM) with a $k-\omega$ turbulence closure. The numerical model is validated against oscillatory flows past a single circular cylinder where the experimental data are available in literature. Then the numerical model is employed to simulate the flow around cylinders in a tandem arrangement. It is found that the distance between the cylinders affects the flow characteristics. Two parallel transverse vortex streets are observed for large distances and two oblique vortex streets for moderate distances. For small distances, only one vortex street can be found. The two cylinders behave like a single bluff body when the distance between the two cylinders is small. The effect of the distance on the force coefficients are investigated in this paper.

KEY WORDS: vortex shedding, oscillatory flow, finite element method

1. Introduction

Offshore pipelines have been widely used in oil and gas engineering to transport oil and gas products. The interaction between a pipeline and waves (or tide) can be approximated by a circular cylinder subjected to an oscillatory flow. Extensive research on hydrodynamic forces and vortex shedding characteristics on an isolated cylinder have been carried out to date due to its relevance to practical engineering problems. Maull et al. (1978), Obasaju (1988) and Williamson (1985) presented the experimental results about the interaction between single cylinder and oscillatory flow. Justesen (1991), Lin (1996), Chew (2002) investigated this problem using numerical methods. However research on flow around a pipeline bundle of two or more cylinders is rather limited.

Zhao et al. (2005) investigated steady flow past two circular cylinders of different diameters at a low Reynolds number. Zhao et al. (2006) also studied steady turbulent flow around two circular cylinders of different diameters. They found that the position of the small cylinder and the gap between two cylinders have profound effects on the hydrodynamic forces on the cylinders and vortex shedding characteristics of the cylinders.

In contrast to the steady flow case, little has been done regarding to oscillatory flow past multiple cylinders. In this study the oscillating flow past two circular cylinders of an identical diameter in tandem (as shown in Figure 5(a)) is investigated numerically. The finite element model proposed by Zhao et al. (2006) is used to simulate the turbulent flow. The numerical model is validated against oscillating flow past a single cylinder before it is applied to simulate the flow around two cylinders. The effects of gap ratios of the two cylinders and the KC numbers on force coefficients and flow characteristics are examined. Five gap ratios (G/D=0.5, 1.0, 2.0, 3.0 and 4.0) and three KC numbers (KC=10, 20 and 28) are calculated. The ratio of the Reynolds number (Re) to the KC number (β) is fixed at a constant of 196.

2. The Problem formulation and Numerical Method

The two-dimensional incompressible Reynolds-Averaged Navier-Stokes equations are used as the governing equations in this study. The non-dimensional equations are written in the Cartesian coordinates as:

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}=-\frac{\partial}{\partial x_i}\left(p+\frac{2}{3}k\right)+\frac{\partial}{\partial x_j}\left(\frac{1}{\mathrm{Re}}\frac{\partial u_i}{\partial x_j}+2\nu_t S_{ij}\right) \tag{1}$$

$$\frac{\partial u_i}{\partial x_i}=0 \tag{2}$$

where x_i ($i=1$ and 2) is the Cartesian coordinate, u_i is the velocity component, p is the pressure, k is the turbulent kinetic energy, Re ($=U_mD/\nu$) is the Reynolds number defined by the amplitude of the oscillating flow velocity U_m and the main cylinder diameter D; ν is the molecular kinetic viscosity, t is time, S_{ij} is the mean stress tensor which is defined as

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{3}$$

The k-ω two-equation model (Wilcox, 1994) is used to determine the turbulent kinetic energy k and the turbulent viscosity ν_t in this study. The equations for k and ω are expressed as

$$\frac{\partial k}{\partial t} + u_j\frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\frac{1}{\mathrm{Re}} + \sigma^*\nu_t\right)\frac{\partial k}{\partial x_j}\right] + P_k - \beta^*\omega k \tag{4}$$

$$\frac{\partial \omega}{\partial t} + u_j\frac{\partial \omega}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\frac{1}{\mathrm{Re}} + \sigma\nu_t\right)\frac{\partial \omega}{\partial x_j}\right] + \alpha\frac{\omega}{k}P_k - \beta\omega^2 \tag{5}$$

where $x_1 = x$, $x_2 = z$, k is the turbulent kinetic energy, $\omega = \varepsilon/(\beta^* k)$ is the specific dissipation of turbulent kinetic energy, ε is the dissipation of turbulent kinetic energy, $P^k = \nu_t\left(\partial u_i/\partial x_j + \partial u_j/\partial x_i\right)\partial u_i/\partial x_j$ is the turbulent kinetic energy production rate. The eddy viscosity ν_t in the k-ω model is calculated by $\nu_t = \alpha^* k/\omega$. The model closure coefficients in Eqs. (4) and (5) are defined as

$$\alpha^* = (1/40 + \mathrm{Re}_T/6)/(1 + \mathrm{Re}_T/6)$$
$$\alpha = (5/9)(1/10 + \mathrm{Re}_T/2.7)/(1 + \mathrm{Re}_T/2.7)(a^*)^{-1}$$
$$\beta^* = (9/100)[5/18 + (\mathrm{Re}_T/8)^4]/[1 + (\mathrm{Re}_T/8)^4]$$
$$\beta = 3/40$$
$$\sigma^* = \sigma = 1/2$$

The finite element method model by Zhao et al. (2006) is used to solve the above Navier-Stokes equations and the turbulent equations. The details about the method can be found in the Zhao et al. (2006) and would not be given in this paper.

3. The Validation of The Numerical Model

The numerical model used in this study is validated against oscillatory flow around a single cylinder before it is applied to simulate the flow around two cylinders and this has been detailed by An et al. (2006). It is found that the vortex shedding regimes, flow characteristics and force coefficients obtained from this numerical method generally agree with the experimental results well. Some of the validation results are given below.

Fig. 1 shows the typical vorticity contours in one period for a single cylinder case at KC=10. Fig. 2 shows the normalized velocities corresponding to the vorticity contours at each instant shown in Fig. 1. The velocity pointing to the right is defined as positive in Fig. 1. It can be seen from Fig. 1 that there is one

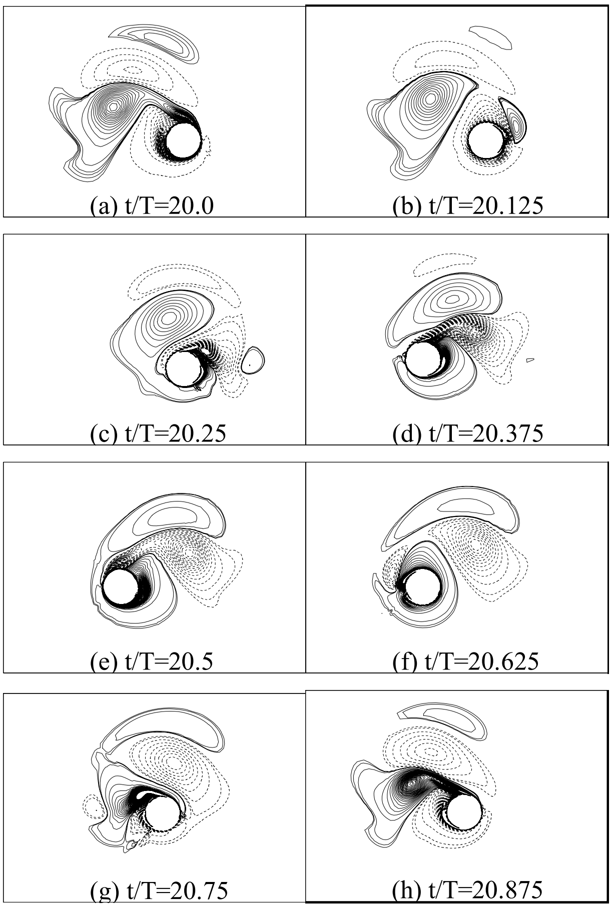

Fig.1 Instantaneous vorticity contours for KC=10 and β=196

vortex shed from the top side of the cylinder in each half of a cycle of the oscillatory flow. The vortices shed from the cylinder formed a vortex street in the direction roughly perpendicular to the direction of oscillatory flow. This kind of wake was called transverse vortex street (Williamson 1985).

Fig. 3 shows the time histories of the in-line force and the lift force on the cylinder respectively. The in-line and the lift forces are normalized by $\rho D U_m^2/2$, where U_m is the velocity amplitude of the oscillatory flow. The oscillation of the lift force is caused by vortex shedding and the flow reversals (Williamson 1985). This means that the number of the peak values (either positive or negative) of the lift force equals the sum of the number of vortex shedding in one period and the number of flow reversals (equals to two in this study). In this case, the lift force has four peaks in one period. The four peaks are caused by two shed vortices and two flow reversals.

4. Oscillatory Flow past Two Circular Cylinders of an Identical Diameter

The numerical model is applied to simulate the oscillatory flow past two circular cylinders with an