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Alternative mixed-integer linear programming models of a maritime inventory routing problem

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ABSTRACT

A single product maritime inventory routing problem is addressed in this paper by exploring the use of continuous and discrete time models. We first present a continuous time model based on time slots for single docks, which is enhanced by reformulating the time assignment constraints. Next, we present a model based on event points to handle parallel docks. A discrete time is also presented based on a single commodity fixed-charge network flow problem (FCNF). All the models are solved for multiple randomly generated instances of different problems to compare their computational efficiency, and to illustrate the solutions obtained.

1. Introduction

Maritime transportation is a major mode of transportation involving 9.6 billion tons in 2013 (UNCTAD, 2014). When one actor or cooperating actors in the maritime supply chain have the responsibility of both the transportation of goods and the inventories at the ports, the underlying planning problem is a maritime inventory routing problem (MIRP). Such problems are complex. However, improvements in the fleet utilization and charge/discharge amounts can translate into large cost reductions. This means that there is great potential and need for research in the area of MIRPs (Agra et al., 2013b).

Many works related to MIRP have been reported in the literatures. Ronen (1983, 1993) published two reviews on ship scheduling and related areas, where different levels and modes were discussed. Christiansen et al. (2004, 2013) reviewed the status and perspectives of ship routing and scheduling. Andersson et al. (2010) surveyed the combined inventory management and routing problem from industrial and modeling aspects, and suggested future research with regard to both further development of the research area and industrial needs. Agra et al. (2013a) reviewed the advances in MIRP, and studied valid inequalities for a single product MIRP to tighten a discrete time model. Surveys by Andersson et al. (2010) and Christiansen and Fagerholt (2009) showed that MIRP has received increasing attention in the last decade. As pointed out by Agra et al. (2013a), the real MIRPs are of high complexity.

There are many works on modeling of MIRPs. Sherali et al. (1999) formulated a mixed-integer programming model based on a discrete time representation for the Kuwait Petroleum Corporation (KPC) problem, and presented an alternative aggregate model that retains the main features of the KPC problem to improve the computational efficiency. Savelsbergh and Song (2008) developed a discrete time model for the inventory routing problem with continuous moves based on the integer multi-commodity flow formulation. Furman et al. (2011) developed a mixed-integer programming formulation based on a discrete time representation for vacuum gas oil routing and inventory management. Song and Furman (2013) introduced a flexible modeling framework for inventory routing problem, which can accommodate various practical features using a discrete time representation. Papageorgiou et al. (2014a) proposed an Arc-Flow MILP formulation for single product MIRP based on discrete-time, and two-stage decomposition algorithms. Papageorgiou and Cheon (2014) presented an approximate dynamic programming method for a class of discrete time based long-horizon MIRP. Bilgen and Ozkarahan (2007) presented an MILP model based on a discrete time representation that integrates blending, loading and transportation decisions simultaneously into one model in order to obtain an optimal solution. Cóccola et al. (2015) presented an MILP-based column generation

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Nomenclature

Sets

D	set of destination ports containing 1 element
N^c	set of consumption ports with indices i and j
N^p	set of production ports with indices i and j
O	set of departure ports containing 1 element
V	set of vessels with index v
K_i	set of time slots for port i with index ki, k_{i1} and k_{im} indicate the first and last slots, respectively
T_i	set of event points for port i with index T_{ik}
T_{dis}	set of time points for discrete time representation with index t_l, t_1 and t_m indicate the first and last points, respectively

Binary variables

o_{vit}	1 if vessel v operating at port i in time period t
of_{vit}	1 if vessel v preparing to operate at port i in time period t
sa_{vijt}	1 if vessel v sailing from i to j in time period t
w_{vik}	1 if vessel v visits port i at time slot k
wa_{vit}	1 if vessel v waiting at port i in time period t
y^s_{vik}	1 if vessel v starts to prepare at port i at time point T_{ik}
y^f_{vik}	1 if vessel v finishes preparation and starts to operate at port i at time point T_{ik}
y^e_{vik}	1 if vessel v finishes operation at port i at time point T_{ik}

Continuous variables

d_{vi}	discharge amount of vessel v at port i
la_{vi}	load of vessel v at port i before operation
q_{vi}	charge amount of vessel v at port i
st_{it}	stock of port i at the beginning of time period t for discrete time model
st^e_{ik}	stock of port i at the end of time slot k
st^f_i	stock of port i at the end of time horizon
st^s_{ik}	stock of port i at the beginning of time slot k
ta_{vi}	arrival time of vessel v at port i
te_{ik}	end time of slot k of port i
tf_{vi}	starting time of vessel v to operate at port i
tl_{vi}	departure of time vessel v from port i
to_{vi}	starting time of vessel v to prepare for operation at port i
ts_{ik}	starting time of slot k of port i
tw_{vi}	time duration of vessel v waits for at port i
T_{ik}	the kth time point of port i
TC, TC'	Total cost

Parameters

a_{vij}	1 indicates that vessel v departs from i to j
Cap_i	dock capacity of port i
Co_{vi}	operation cost of vessel v at port i
Csf_v	fixed cost of vessel v for sailing
Csv_v	coefficient of variable cost of vessel v for sailing
Cw_{vi}	coefficient of waiting cost of vessel v at port i
dr_{vit}	discharge rate of vessel v at port i for discrete time model
la^{lo}_v	lowerbound of vessel v's load
la^{up}_v	upperbound of vessel v's load
P_i	production rate of port i
qr_{vit}	charge rate of vessel v at port i for discrete time model
Q_i	consumption rate of port i
rv_{vi}	charge/discharge rate of vessel v at port i
st^{lo}_i	lowerbound of stock of port i
st^{up}_i	upperbound of stock of port i
Td_{vij}	transportation time between port i and j for vessel v for discrete time model
tfi_{vi}	preparation time for charging/discharging of vessel v at port i
TH	time horizon
$TimeWUp_v$	upperbound of vessel v's waiting time
tsa_{vij}	transportation time between port i and j for vessel v for continuous time model, in days

strategy for managing large-scale maritime distribution problems which regards to time windows for vessels visiting ports.

Al-Khayyal and Hwang (2007) studied inventory constrained maritime routing and scheduling for multi-commodity liquid bulk. They defined a *position* (i,m), where i denotes a port, and m is the arrival number of the vessels at that port within the planning horizon. Then, they formulated the continuous time constraints of different arrivals within one port and different positions related to the same vessel. Li et al. (2010) addressed an inventory service problem in which a chemical MNC uses a fleet of multi-parcel ships with dedicated compartments to move multiple chemicals continuously among its internal and external production and consumption sites. They presented a MILP model similar to that of Al-Khayyal and Hwang (2007) at an operational level with finer granularity. Siswanto (2011) presented a variation of Al-Khayyal and Hwang's model in which he relaxed the problem to consider an assignment of multi-undedicated compartments to products.

Goel et al. (2015) proposed two constraint programming models for LNG ship scheduling and inventory management. Agra et al. (2013b) presented an MILP model based on continuous time for a short sea fuel oil distribution problem, and considered several strategies to improve its expression and solution through tighter bounds, an arc-load flow and multi-commodity reformulation and including valid inequalities. Papageorgiou et al. (2014b) presented a detailed description of a particular class of deterministic single product MIRPs called deep-sea MIRPs with inventory tracking at every port, and introduced a core model and gave an excellent review of the existing works.

As pointed out by Sherali and Al-Yakoob (2005a,b), it is possible to solve small practical instances using commercial solvers. However, it is difficult to solve large scale problems due to the large number of binary variables. A computationally efficient model is critical for solving such problems. Furthermore, a discrete time model is generally much larger than continuous time models. Since most of the works have focused on discrete time models, we explore in this paper the potential of continuous time models.

A single product MIRP problem with one actor is studied in this paper. There are two categories of ports: production ports and consumption ports. Each port has an inventory with a certain capacity. Vessels transport products from production ports to consumption ports according to the fixed routings to ensure that the inventories of ports neither exceed given capacities, nor lie below given safety levels. The ports have given deterministic production/consumption rates and dock capacities. The vessels have given capacities, speeds, preparation times and charge/discharge rates. The objective is to determine the times that vessels visit the ports, and the charged/discharged amounts of vessels at each port in order to minimize total cost while satisfying the inventory constraints.