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Resource constrained project scheduling problem with setup times after preemptive processes

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ABSTRACT

In this paper, the preemptive resource constrained project scheduling problem with set up times is investigated. In this problem, a fixed setup time is required to restart when an process is preempted. The project contains activities inter-related by finish to start type precedence relations with a time lag of zero, which require a set of renewable resources. The problem formed in this way is an NP-hard. A mixed integer programming model is proposed for the problem and a parameters tuned meta-heuristic namely genetic algorithm is proposed to solve it. To evaluate the validation and performance of the proposed algorithm a set of 100 test problems is used. Comparative statistical results show that the proposed algorithm is efficiently capable to solve the problem.

1. Introduction

The resource constrained project scheduling problem (RCPSP) is a known optimization problem which is NP-hard (Blazewicz et al., 1983). The objective of RCPSP is to minimize the makespan of the project while the renewable resources availabilities are considered given. In the literature there are several exact methods and heuristics that solve the RCPSP (Pritsker et al., 1969; Christofides et al., 1987; Choi et al., 2004; Hartmann and Kolisch, 2006; Zhang et al., 2006; Hartmann and Briskorn, 2010; Jairo et al., 2010; Agarwal et al., 2011; Fang and Wang, 2012; Koné, 2012; Kyriakidis et al., 2012; Paraskevopoulos et al., 2012; Jia and Seo, 2013; Kopanos et al., 2014).

In the basic project scheduling problems it is assumed that each activity once started, will be executed until its completion. Preemptive project scheduling problem refers to the scheduling problem which allows activities to be preempted at any discrete time instance and restarted later. There are some solution methods for the preemptive project scheduling problems in the literature (Kaplan, 1988; Demeulemeester and Herroelen, 1996; Buddhakulsomsiri and Kim, 2006; Damay et al., 2007; Ballestin et al., 2008; Vanhoucke and Debels, 2008; Van Peteghem and Vanhoucke, 2010; Afshar-Nadjafi, 2014; Haouari et al., 2014).

While setup times have been widely studied in machine scheduling (Anglani et al., 2005; Roshanaei et al., 2009; Liao et al., 2012; Nagano et al., 2012) the literature concerning setup times in the context of project scheduling is scant. Generally, in project scheduling a setup has been defined as a preparation of all requirements for execution an activity. The time needed for this preparation is then called a setup time. In the area of project scheduling, the assumption that the duration of an activity reflects both setup and processing times has also been made throughout the years. This common assumption can be justified as long as setup times are relatively small in comparison to processing times. However, in the case which activities require some considerable setup times, modeling and solving such a problem as a classical RCPSP, especially in the preemptive case may lead to poor solutions (Kolisch, 1995).

Motivation of this paper is to show how to model the setup times in the preemptive RCPSP. First, a mixed integer programming model is developed for the preemptive resource constrained project scheduling problem with setup times. This problem is called PRCPSP-ST. This model is not considered in the past literature. Second, a new efficient parameter-tuned algorithm namely genetic algorithm is developed to solve it due to NP-hardness of the problem. Finally, the effectiveness of the proposed method to solve the PRCPSP-ST is evaluated and effect of setup time on makespan of the project is analyzed.

Reminder of the paper is organized as follows: Section 2 describes the problem PRCPSP-ST and mixed integer formulation for it. Section 3 explains the steps of the proposed algorithm to solve the problem. Section 4 contains the computational results and

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performance evaluation of the proposed algorithm. Finally, Section 5 concludes the paper.

2. Problem description

The preemptive resource constrained project scheduling problem with setup times (PRCPSP-ST) involves the scheduling of project's activities on a set K of renewable resource types. Each activity i is performed in a single mode with deterministic duration of d_i. Each activity i requires r_{ik} units of renewable resource type k (k = 1, ..., K) during each time unit of its execution. For each renewable resource type k, the availability R_k is constant throughout the project horizon. In sequent, assume a project represented in activity on node, *AON*, format by a directed graph $G = \{N, A\}$ where the set of nodes, N, represents activities and the set of arcs, A, represents finish to start precedence relations with a time-lag of zero. The preempt-able activities are numbered from the dummy start activity 0 to the dummy end activity $n+1$ and are topologically ordered, i.e. each successor of an activity has a larger activity number than the activity itself. Once an activity i is preempted, a setup time ST_i is required to restart the activity. The following assumptions are considered in the PRCPSP-ST:

- The activities can be preempted in discrete time points.
- The number of preemptions for an activity is not limited.
- A setup time is required to start an activity after preempted.
- Initial setup time of an activity is included in its duration.
- Setup times are deterministic and schedule-independent.
- Setups are inseparable, i.e. an activity is started immediately after its setup is finished.
- Setups occurred after preemption require the same amount of renewable resources with that of when the activity is processing.
- All parameters are integers.

The objective of the PRCPSP-ST is to schedule a number of activities, in order to minimize makespan of the project. A schedule S is defined by a vector of activities finish (start) times and is said to be feasible if all precedence relations and renewable resources constraints are satisfied. Let $f_{i,j}$ denotes the finish time of jth unit of activity i. The unit of an activity j can be defined as the smallest discrete segment of the activity, i.e. an hour, a day, a week, etc. In order to ease the formulation, $f_{i,0}$ can be used to denote start time of activity i. By defining binary decision variables x_{ij} which specify whether jth unit ($1 \le j \le d_i - 1$) of an activity i is preempted or not, PRCPSP-ST can be conceptually formulated as follows:

$$\min\ C_{\max} = f_{(n+1),0} \tag{1}$$

Subject to:

$$f_{i,d_i} \le f_{j,0}; \quad \forall (i,j) \in A \tag{2}$$

$$f_{i,j-1} + 1 \le f_{i,j} - x_{i,(j-1)}(1 + ST_i); \quad i = 0, 1, \ldots, n+1; \quad j = 1, \ldots, d_i \tag{3}$$

$$f_{i,j} - x_{i,(j-1)}(1 + ST_i) \le f_{i,j-1} + 1 + Mx_{i,(j-1)}; \quad i = 0, 1, \ldots, n+1; \quad j = 1, \ldots, d_i \tag{4}$$

$$f_{0,0} = 0 \tag{5}$$

$$\sum_{i \in S_t} r_{ik} \le R_k; \quad k = 1, \ldots, K; \quad t = 1, \ldots, f_{(n+1),0} \tag{6}$$

$$x_{i,j} \in \{0, 1\}, f_{i,j} \in \text{Int}; \quad i = i = 0, 1, \ldots, n+1; \quad j = 0, 1, \ldots, d_i \tag{7}$$

The objective in Eq. (1) is to minimize the makespan of the project. Eq. (2) denotes the finish to start zero time lag precedence relations. Eqs. (3) and (4) guarantee that setup time should be taken into account if an activity is preempted. Parameter M is a considerably positive constant. However, Eqs. (3) and (4) preserve the relation between x_{ij} and f_{ij}. Eq. (5) specifies that start dummy activity 0 should be started at time 0. Constraint set in Eq. (6) take care of the renewable resources limitations. It represents renewable resource constraints for every resource type k by considering for every time instant t for all activities i such that the activity is in progress in period t. S_t denotes the set of activities which are in progress or their setups are in progress at time interval $[t-1, t]$. This constraint set is true logically, however, it cannot be solved directly because there is no easy way to translate the set S_t. Eq. (7) specifies that the decision variables f_{ij} are integers, while x_{ij} are binary.

In continue we developed an improved mathematical formulation for PRCPSP-ST based on following decision variables:

x_{ivt} 1, if vth unit of activity i is finished at time t
0, otherwise (binary decision variable)
y_{ivt} 1, if vth unit ($1 \le v \le d_i - 1$) of activity i is preempted at time t
0, otherwise (binary decision variable)
z_{ivt} 1, if setup of v^{th} unit ($2 \le v \le d_i$) of activity i is in progress at time interval $[t-1, t]$
0, otherwise (binary decision variable)

It is clear that an activity with duration of 0 is never in progress and thus does not have a corresponding decision variable which is set to 1. This problem, however, can be easily overcome: the dummy start and end activity are assigned a dummy mode with duration of 1. Also, the other parameters for dummy modes are assumed 0. All other activities with zero duration can be eliminated, provided that the corresponding precedence relations are adjusted appropriately. The resulting schedule may be transferred into a schedule for the original problem by removing the dummy start and end activity, and one time unit left shifting. Using the above notation, the PRCPSP-ST can be mathematically formulated as follows, where $EST(i)$ and $LST(i)$ denote the earliest start time and the latest start time of activity i, respectively.

$$\text{Min} = \sum_{t=EST(n+1)+1}^{LST(n+1)+1} t\, x_{(n+1)1t} \tag{8}$$

The objective in Eq. (8) is to minimize the makespan of the project.

$$\sum_{t=EST(i)+1}^{LST(i)+1} t\, x_{id_it} \le \sum_{t=EST(j)+1}^{LST(j)+1} t\, x_{j1t} - 1, \quad \text{for} \quad (i,j) \in A \tag{9}$$

Eq. (9) denotes the finish to start zero time lag precedence relations constraints.

$$\sum_{t=EST(i)+(v-1)}^{LST(i)+(v-1)} tx_{i(v-1)t} + 1 \le \sum_{t=EST(i)+v}^{LST(i)+v} tx_{ivt} - \left[\sum_{t=EST(i)+(v-1)}^{LST(i)+(v-1)} yi_{(v-t)t}\right] \times (1 + ST_i) \le \sum_{t=EST(i)+(v-1)}^{LST(i)+(v-1)} tx_{i(v-1)t} + 1 + M \quad \text{for} \quad \{i \in Nd_i \ne 1\}, \quad v = 2, \ldots, d_i \tag{10}$$

Eq. (10) guarantees that setup time should be taken into account if activity is preempted. Parameter M is a considerably positive constant.

$$x_{ivt} \le x_{x_{i(v+t)(t+1)}} + y_{ivt} \le 1 \quad \text{for} \quad \{i \in Nd_i \ne 1\}, \quad v = 1, \ldots, d_i - 1, \quad t = EST(i) + v, \ldots, LST(i) + v \tag{11}$$