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Economic model predictive control for inventory management in supply chains

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ABSTRACT

In this paper, we propose economic model predictive control with guaranteed closed-loop properties for supply chain optimization. We propose a new multiobjective stage cost that captures economics as well as risk at a node, using a weighted sum of an economic cost and a tracking stage cost. We also demonstrate integration of scheduling with control using a supply chain example. We integrate a scheduling model for a multiproduct batch plant with a control model for inventory control in a supply chain. We show recursive feasibility of such integrated control problems by developing simple terminal conditions.

1. Introduction

Control theory for inventory management in supply chains is an important area of research (Ortega & Lin, 2004; Sarimveis, Patrinos, Tarantilis, & Kiranoudis, 2008). Over the past decade, rolling horizon optimization for inventory management has been studied (Braun, Rivera, Flores, Carlyle, & Kempf, 2003; Dunbar & Desa, 2007; Kempf, 2004; Maestre, Muñoz de la Peña, & Camacho, 2009, 2011; Mestan, Türkay, & Arkun, 2006; Perea López, Ydstie, & Grossmann, 2003; Seferlis & Giannelos, 2004). An important consideration in process control is the stability of the closed-loop under the proposed control. In Lin, Wong, Jang, Shieh, and Chu (2004), Venkateswaran and Son (2005), and Hoberg, Bradley, and Thonemann (2007), the closed-loop stability for classical control theory applied to inventory management was studied. However, the stability of rolling horizon optimization methods for inventory management has not been studied extensively. Model predictive control (MPC) is a rolling horizon optimization based control algorithm with guaranteed stability properties. Although, stability theory for MPC is a fairly mature field (Rawlings & Mayne, 2009), most implementations of MPC for supply chain do not consider stability. In Subramanian, Rawlings, and Maravelias (2013), we proposed centralized and cooperative MPC with stability and asymptotic convergence guarantees for a supply chain tracking inventories to its setpoint. Our focus in this paper is to use recent developments in economic MPC (Amrit, Rawlings, & Angeli, 2011; Diehl, Amrit, & Rawlings, 2011) to develop a closed-loop stable model predictive controller for supply chains in which we directly optimize the economic cost of operating the supply chain.

This paper is organized as follows. In Section 2, we briefly describe the main results of economic MPC. In Section 3, we derive the state space model for a two-node supply chain. In Section 3.1, we show the results for economic MPC on a simple two-node supply chain. In Section 3.2, we propose a novel multiobjective MPC formulation that accounts for supply chain economics as well as risk. In Section 4 we implement MPC for a multiechelon, multiproduct supply chain example. In Section 5, we use periodic terminal condition ideas from Subramanian, Maravelias, and Rawlings (2012) to integrate scheduling with inventory control. Finally, we present our conclusions in Section 6.

2. Economic MPC

Note. We state only important economic MPC stability results in this section. The interested reader can refer to Amrit et al. (2011) and Diehl et al. (2011) for details.

Model. We consider the following linear model

$$x^+ = Ax + Bu + B_d d \qquad (1)$$

in which $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the manipulated input and $d \in \mathbb{R}^d$ is the disturbance to the system. We develop economic MPC theory for the nominal disturbance, denoted by d_s. We assume that the system (A, B) is stabilizable.

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Constraints. The states and inputs are constrained as follows:

$$x \in \mathbb{X} \quad u \in \mathbb{U} \tag{2}$$

Stage cost. The economic cost for implementing input u from state x is given by $\ell(x, u)$.

Optimal steady state. We define the steady-state problem for the nominal demand d_s as follows:

$$\min_{x,u} \ell(x, u) \qquad \text{s.t. } x = Ax + Bu + B_d d_s, \quad x \in \mathbb{X}, u \in \mathbb{U} \tag{3}$$

The optimal steady state is denoted by $(x_s, u_s\,; d_s)$

We make the following assumptions:

Assumption 1. The constraint set $\mathbb{X}$ is convex and closed. The constraint set $\mathbb{U}$ is convex and compact. The optimal steady state $(x_s, u_s\,; d_s)$ is such that $x_s \in \mathbb{X}$ and $u_s \in \mathbb{U}$

Assumption 2. There exists $(x_s, u_s\,; d_s)$ and λ_s so that

(a) $(x_s, u_s\,; d_s)$ is a unique solution of (3).
(b) The multiplier λ_s is such that $(x_s, u_s\,; d_s)$ uniquely solves (4)

$$\min_{x,u} \ell(x, u) + \lambda_s'[x - (Ax + Bu + B_d d_s)] \quad \text{s.t. } x \in \mathbb{X}, u \in \mathbb{U} \tag{4}$$

(c) The system $x^+ = Ax + Bu + B_d d_s$ is strictly dissipative with respect to the supply rate $s(x, u) = \ell(x, u) - \ell(x_s, u_s)$ and storage function $\lambda(x) = \lambda'_s x$. That is, there exists a positive definite function $\rho(\cdot)$ such that for all $(x, u) \in \mathbb{X} \times \mathbb{U}$

$$\lambda_s'(Ax + Bu + B_d d_s - x) \leq -\rho(x - x_s) + s(x, u) \tag{5}$$

Assumption 3 *(Basic stability assumption).* There exists a convex, compact terminal region $\mathbb{X}_f \subseteq \mathbb{X}$, containing the point x_s and a control law $\kappa_f : \mathbb{X}_f \to \mathbb{U}$ and a function $V_f : \mathbb{X}_f \to \mathbb{R}$ such that the following holds for all $x \in \mathbb{X}_f$

$$V_f(Ax + B\kappa_f(x) + B_d d_s) \leq V_f(x) - \ell(x, \kappa_f(x)) + \ell(x_s, u_s) \tag{6}$$

$$Ax + B\kappa_f(x) + B_d d_s \in \mathbb{X}_f \tag{7}$$

We first define the terminal constraint MPC problem

$$\begin{aligned} \mathbb{P}_N(x; d_s): \quad & \min_{\mathbf{u}} V_N(\mathbf{u}; x) \\ & \text{s.t. } x(0) = x \\ & x(j+1) = Ax(j) + Bu(j) + B_d d_s, \quad j \in \mathbb{I}_{0:N-1} \\ & x(j) \in \mathbb{X} \qquad j \in \mathbb{I}_{0:N-1} \\ & u(j) \in \mathbb{U} \qquad j \in \mathbb{I}_{0:N-1} \\ & x(N) = x_s \end{aligned} \tag{8}$$

in which the cost function $V_N(\mathbf{u}\,; x)$ is the sum of stage costs

$$V_N(\mathbf{u}; x) = \sum_{j=0}^{N-1} \ell(x(j), u(j)) \tag{9}$$

The control horizon is denoted by N, the input sequence by $\mathbf{u} = (u(0), u(1), \ldots, u(N-1))$ and the symbol $\mathbb{I}_{k:l}$ stands for the set $\{k, k+1, \ldots, l\}$.

The control law $\kappa(x)$ is the first input in the optimal solution $\mathbf{u}^0(x)$ to optimization problem (8). The admissible region $\mathcal{X}_N$ is given by

$$\mathcal{X}_N := \{x \in \mathbb{X} \mid \exists \mathbf{u} \in \mathbb{U}^N, \text{ s.t.} \quad (8) \quad \text{is feasible}\}$$

The following is the exponential stability theorem for economic MPC that solves problem (8) online (Diehl et al., 2011).

Theorem 4 (Lyapunov function with terminal constraint). *Let the system (A, B) be stabilizable. Let Assumptions 1 and 2 hold. Then the steady-state solution of the closed-loop system $x^+ = Ax + B\kappa(x) + B_d d_s$ is asymptotically stable with $\mathcal{X}_N$ as the region of attraction. The Lyapunov function is*

$$\tilde{V}(x) := V_N^0(x) + \lambda_s'[x - x_s] - N\ell(x_s, u_s)$$

in which $V_N^0(x)$ is the optimal cost function of (8)

We also define the terminal region/penalty MPC problem as follows:

$$\begin{aligned} \mathbb{P}_N(x; d_s): \quad & \min_{\mathbf{u}} V_N(\mathbf{u}; x) \\ & \text{s.t. } x(j+1) = Ax(j) + Bu(j) + B_d d_s, \quad j \in \mathbb{I}_{0:N-1} \\ & x(j) \in \mathbb{X} \qquad j \in \mathbb{I}_{0:N-1} \\ & u(j) \in \mathbb{U} \qquad j \in \mathbb{I}_{0:N-1} \\ & x(N) \in \mathbb{X}_f \end{aligned} \tag{10}$$

in which the cost function $V_N(\mathbf{u}\,; x)$ is

$$V_N(\mathbf{u}; x) = \sum_{j=0}^{N-1} \ell(x(j), u(j)) + V_f(x(N)) \tag{11}$$

Note that in contrast to objective function (9), we modify the cost function in the terminal penalty formulation by adding a terminal penalty $V_f(x(N))$. Similarly, in the optimization problem (10), the terminal constraint is replaced by the terminal region. The terminal region $\mathbb{X}_f$ and the terminal penalty $V_f(\cdot)$ are chosen to satisfy Assumption 3. We use $\kappa(x)$ to denote the first input in the optimal input sequence to problem (10). The admissible region is defined as the set of states for which (10) admits a feasible solution.

The following is the theorem for exponential stability of economic MPC using the terminal region/penalty formulation (Amrit et al., 2011).

Theorem 5 (Lyapunov function with terminal penalty). *Let the system (A, B) be stabilizable. Let Assumptions 1, 2 and 3 hold. Then the steady-state solution of the closed-loop system $x^+ = Ax + B\kappa(x) + B_d d_s$ is asymptotically stable with $\mathcal{X}_N$ as the region of attraction. The Lyapunov function is*

$$\tilde{V}(x) := V_N^0(x) - N\ell(x_s, u_s) - \lambda'_s(x_s) - V_f(x_s)$$

in which $V_N^0(x)$ is the optimal value function of (10)

3. Two-node, single-product supply chain

Fig. 1 shows a two-stage, single-product supply chain with a retailer and a manufacturer. The manufacturing delay as well as the shipment delay is 2 time units. The retailer responds to the customer demand Dm (nominal demand d_s) by shipping S_1 units to the customer and ordering O_1 units to the manufacturer. These decisions are based on the retailer "states": the inventory at the retailer, Iv_1, and the backorder at the retailer, BO_1. The inventory and backorder balance equations are the dynamics of the retailer states and can be written as:

$$\text{Iv}_1(k+1) = \text{Iv}_1(k) + S_2(k-2) - S_1(k)$$

$$\text{BO}_1(k+1) = \text{BO}_1(k) - S_1(k) + \text{Dm}(k)$$

in which $S_2(k-2)$ is the shipment made by the manufacturer two time periods ago.

Similarly, the manufacturer responds to retailer orders O_1 by making shipments S_2 and production O_2. The dynamics for the manufacturer is:

$$\text{Iv}_2(k+1) = \text{Iv}_2(k) + O_2(k-2) - S_2(k)$$