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Stochastic optimal control model for natural gas networks

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ABSTRACT

We present a stochastic optimal control model to optimize gas network inventories in the face of system uncertainties. The model captures detailed network dynamics and operational constraints and uses a weighted risk-mean objective. We perform a degrees-of-freedom analysis to assess operational flexibility and to determine conditions for model consistency. We compare the control policies obtained with the stochastic model against those of deterministic and robust counterparts. We demonstrate that the use of risk metrics can help operators to systematically mitigate system volatility. Moreover, we discuss computational scalability issues and effects of discretization resolution on economic performance.

1. Introduction

Consider a gas network with links comprising of long pipelines and nodes consisting of junction points and compressors. Gas is withdrawn from the network at a set of demand nodes and make-up gas is brought into the system through a set of supply nodes. In a real-time environment, the system operator must balance the network to satisfy demand flows and delivery pressures at all times. To achieve this balance, compressors are operated to coordinate buildup and release of inventory inside the pipes. This procedure, called "line-pack management" (Rachford & Carter, 2000), consists on determining dynamic operating policies for the compressors to balance supply, inventory, and demand. The policies must respect compression limits and minimize compressor power or fuel. One of the key issues arising in operations is that demand profiles cannot be predicted with full certainty and thus inventory must be built up, *in advance*, to ensure that enough capacity is available to satisfy a range of possible future scenarios. Uncertainty in gas pipeline operations is becoming an increasing concern as the power grid adopts larger amounts of intermittent weather-driven resources, because gas-fired power plant units are typically used to balance supply at short notice (Liu, Shahidehpour, Fu, & Li, 2009; Rachford & Carter, 2000).

Optimization of gas networks has been performed in diverse studies. These studies differ in the decision setting and physical models used. Optimization models for mid-term planning and contracting purposes do not require information about line-pack dynamics so steady-state models are appropriate. O'Neill, Williard, Wilkins, and Pike (1979) present a steady-state transmission model. De Wolf and Smeers (2000) develop an extension of the simplex method to solve models with this structure. An optimal design model for pipes diameters is proposed by the same authors in De Wolf and Smeers (1996). Martin, Möller, and Moritz (2006) present a steady-state nonlinear transmission model that allows for hybrid (on/off) decisions (a mixed-integer nonlinear optimization model) and develop strategies to approximate nonlinear terms using piecewise linear functions, thus enabling the use of mixed-integer linear programming solvers.

For real-time operations, system dynamics must be captured in order to ensure feasible and implementable policies. Moritz (2007) presents a mixed-integer optimal control model with detailed conservation and momentum equations, network balances, and hybrid valve and compressor components. As the author acknowledges, however, computational limitations forced her to consider conservation and momentum equations in simplified form, by defining only inlet and output points. This is equivalent to discretizing the underlying partial differential equations (PDEs) using two discretization points placed at the boundary nodes. Ehrhardt and Steinbach (2005) present a nonlinear continuous optimal control model in which compressor policies are optimized to satisfy demands and minimize compressor fuel. A full space-time discretization of the PDEs is performed and a sequential quadratic programming algorithm is used for the solution of the resulting nonlinear programming (NLP) problem. Steinbach (2007) proposed the use of an interior point algorithm to solve the NLP and proposed a strategy to exploit the underlying linear algebra structure. These studies focused on computational performance, with limited modeling and economic performance analysis. Baumrucker and Biegler (2010) present an optimal control formulation allowing for hybrid

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behavior arising from flow reversals. The authors cast the problem as a mathematical program with equilibrium constraints and analyze the effect of different electricity price structures on economic performance.

None of the real-time optimization models available in the literature accounts for uncertainty, with the exception of the work of Carter and Rachford (2003). In their work, they present a detailed discussion of uncertainties prevailing in real-time operations and discuss the benefits of using stochastic optimization formulations to manage line-pack inventory. The authors provide a sound physical analysis of the resulting optimal policies; however, they do not report the model and the solution strategy used.

In this work, we present a detailed stochastic optimal control model that considers conservation and momentum equations, typical operational constraints, and uncertainty in demands. We perform a degrees-of-freedom (DOF) analysis to verify the consistency of the model and we use this analysis to derive consistent initial conditions and nonanticipacity constraints. In addition, we propose to incorporate a risk metric into the objective function to mitigate cost variance and system volatility. Using a computational study, we demonstrate the benefits obtained with stochastic formulations against deterministic and robust counterparts and we discuss the effects of discretization mesh resolution on economic performance.

The paper is structured as follows. In Section 2 we present the physical model for the pipelines, network, and compressors. In Section 3 we present the DOF analysis to characterize the differential and algebraic equation (DAE) system and provide conditions to achieve model consistency. In Section 4 we formulate the stochastic optimal control model by defining the objective function, operational constraints, initial conditions, and nonanticipativity constraints. In Section 5 we present a computational study to demonstrate the benefits of the stochastic model over a range of different formulations and we discuss computational issues. The paper closes in Section 6 with concluding remarks and directions of future work. The model nomenclature as well as variables and parameter units are presented in Appendix A.

2. Physical model

In this section, we present the conservation and momentum equations governing the dynamics of each pipeline in the network as well as the equations describing the network interconnections. Nomenclature, physical units, and typical values for all variables and parameters are given in Appendix A.

2.1. Conservation, momentum, and network

We assume an isothermal gas flow through a horizontal pipe and define a set $\mathcal{L}$ of pipes or links. The conservation and momentum equations for a given link $\ell \in \mathcal{L}$ are given by the following set of PDEs (Osiadacz, 1984; Van Deen & Reintsema, 1983):

$$\frac{\partial \rho_\ell(\tau, x, \omega)}{\partial \tau} + \frac{\partial(\rho_\ell(\tau, x, \omega)\nu_\ell(\tau, x, \omega))}{\partial x} = 0 \tag{2.1a}$$

$$\frac{\partial(\rho_\ell(\tau, x, \omega)\nu_\ell(\tau, x, \omega))}{\partial \tau} + \frac{\partial p_\ell(\tau, x, \omega)}{\partial x} = -\frac{\lambda_\ell}{2D_\ell}\rho_\ell(\tau, x, \omega)\nu_\ell(\tau, x, \omega)|\nu_\ell(\tau, x, \omega)|. \tag{2.1b}$$

Here, $\tau \in \mathcal{T} := [0, T]$ is the time dimension with final time T (planning horizon), and $x \in \mathcal{X}_\ell := [0, L_\ell]$ is the axial dimension with length L_ℓ. We also define a set of scenarios $\omega \in \Omega := \{1..N_\Omega\}$. The link diameters are denoted as D_ℓ and the friction coefficients are denoted as λ_ℓ. The states of the link are the gas density $\rho_\ell(\tau, x, \omega)$, the gas speed $\nu_\ell(\tau, x, \omega)$, and the gas pressure $p_\ell(\tau, x, \omega)$. The transversal area A_ℓ, volumetric flow $q_\ell(\tau, x, \omega)$, and mass flow $f_\ell(\tau, x, \omega)$ are given by

$$A_\ell = \frac{1}{4}\pi D_\ell^2 \tag{2.2a}$$

$$q_\ell(\tau, x, \omega) = \nu_\ell(\tau, x, \omega)A_\ell \tag{2.2b}$$

$$f_\ell(\tau, x, \omega) = \rho_\ell(\tau, x, \omega)\,\nu_\ell(\tau, x, \omega)A_\ell. \tag{2.2c}$$

For an ideal gas, pressure and density are related as follows:

$$\frac{p_\ell(\tau, x, \omega)}{\rho_\ell(\tau, x, \omega)} = c^2. \tag{2.3}$$

Here, c is the gas speed of sound defined in Appendix A.[1] We transform (2.1a) and (2.1b) into a more convenient form in terms of mass flow and pressure by using (2.3) and (2.2a)–(2.2c):

$$\frac{\partial p_\ell(\tau, x, \omega)}{\partial \tau} + \frac{c^2}{A_\ell}\frac{\partial f_\ell(\tau, x, \omega)}{\partial x} = 0 \tag{2.4a}$$

$$\frac{1}{A_\ell}\frac{\partial f_\ell(\tau, x, \omega)}{\partial \tau} + \frac{\partial p_\ell(\tau, x, \omega)}{\partial x} = -\frac{\lambda_\ell \rho_\ell(\tau, x, \omega)}{2D_\ell}\frac{f_\ell(\tau, x, \omega)}{\rho_\ell(\tau, x, \omega)A_\ell}\left|\frac{f_\ell(\tau, x, \omega)}{\rho_\ell(\tau, x, \omega)A_\ell}\right|. \tag{2.4b}$$

Substituting (2.3) and (2.2a) in (2.4b) and performing some manipulations, we obtain the more compact form,

$$\frac{\partial p_\ell(\tau, x, \omega)}{\partial \tau} = -\frac{c^2}{A_\ell}\frac{\partial f_\ell(\tau, x, \omega)}{\partial x} \tag{2.5a}$$

$$\frac{1}{A_\ell}\frac{\partial f_\ell(\tau, x, \omega)}{\partial \tau} = -\frac{\partial p_\ell(\tau, x, \omega)}{\partial x} - \frac{8\,\lambda_\ell\, c^2}{\pi^2 D_\ell^5}\frac{f_\ell(\tau, x, \omega)|f_\ell(\tau, x, \omega)|}{p_\ell(\tau, x, \omega)}. \tag{2.5b}$$

For numerical purposes, we define scaled flows $f_\ell(\tau, x, \omega) \leftarrow \alpha_f f_\ell(\tau, x, \omega)$ and pressures $p_\ell(\tau, x, \omega) \leftarrow \alpha_p p_\ell(\tau, x, \omega)$, where α_f and α_p are scaling factors. Scaling (2.5a) and (2.5b) and rearranging, we obtain

$$\frac{\partial p_\ell(\tau, x, \omega)}{\partial \tau} = -c_{1,\ell}\frac{\partial f_\ell(\tau, x, \omega)}{\partial x}, \quad \ell \in \mathcal{L}, \tau \in \mathcal{T}, x \in \mathcal{X}_\ell, \omega \in \Omega \tag{2.6a}$$

$$\frac{\partial f_\ell(\tau, x, \omega)}{\partial \tau} = -c_{2,\ell}\frac{\partial p_\ell(\tau, x, \omega)}{\partial x} - c_{3,\ell}\frac{f_\ell(\tau, x, \omega)|f_\ell(\tau, x, \omega)|}{p_\ell(\tau, x, \omega)}, \quad \ell \in \mathcal{L}, \tau \in \mathcal{T}, x \in \mathcal{X}_\ell, \omega \in \Omega, \tag{2.6b}$$

where the constants $c_{1,\ell}$, $c_{2,\ell}$, and $c_{3,\ell}$ are defined in Appendix A.

We now consider a network with a set $\mathcal{N}$ of nodes, a set $\mathcal{L}$ of links, a set $\mathcal{S}$ of supply flows, and a set $\mathcal{D}$ of demand flows. For each node $n \in \mathcal{N}$ we define the set of inlet and outlet links, $\mathcal{L}_n^{in} := \{\ell \mid rec(\ell) = n\}$, $\mathcal{L}_n^{out} := \{\ell \mid snd(\ell) = n\}$. Here, $rec(\ell)$ is the receiving node of link ℓ and $snd(\ell)$ is the sending node of link ℓ. We define $dem(j)$ as the node at which the demand flow $d_j(\tau, \omega)$ is located and $sup(i)$ as the node at which the supply flow $s_i(\tau, \omega)$ is located. Accordingly, we define the sets For $\mathcal{S}_n := \{j \in \mathcal{S} \mid \sup(j) = n\}$ and $\mathcal{D}_n := \{j \in \mathcal{D} \mid dem(j) = n\}$ for each node $n \in \mathcal{N}$.

[1] Expression (2.3) is derived by noticing that the gas speed of sound for any gas is given by $c^2 = B/\rho$ where B is the bulk modulus and ρ is the gas density. We also have that $B = -Vdp/dV$ where p is the gas pressure and V is the volume. When a sound travels through an ideal gas, the rapid compressions and expansions can reasonably be expected to be adiabatic and thus we have $pV^\gamma = C$ where γ is the adiabatic constant and C is an arbitrary constant. We take the derivative of $p = V^{-\gamma}C$ with respect to V and plug the resulting expression in the expression for B. We then make use of the gas law $p/\rho = zRT_{gas}/M$ to obtain the result (2.3).