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Deterministic optimization of the thermal Unit Commitment problem: A Branch and Cut search

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ABSTRACT

This paper proposes a novel deterministic optimization approach for the Unit Commitment (UC) problem, involving thermal generating units. A mathematical programming model is first presented, which includes all the basic constraints and a set of binary variables for the on/off status of each generator at each time period, leading to a convex mixed-integer quadratic programming (MIQP) formulation. Then, an effective solution methodology based on valid integer cutting planes is proposed, and implemented through a Branch and Cut search for finding the global optimal solution. The application of the proposed approach is illustrated with several examples of different dimensions. Comparisons with other mathematical formulations are also presented.

1. Introduction

The increasing electricity demand motivates the need to study different operational alternatives for planning power generation by integrating conventional generation sources with renewables, while ensuring profitability (Verhaegen, Meeus, Delvaux, & Belmans, 2007; del Río, 2011; Ventosa, Baíllo, Ramos, & Rivier, 2005; Xiao, Hodge, Pekny, & Reklaitis, 2011); as well as, ways of improving the energy efficiency of existing power systems (Siirola & Edgar, 2012). Furthermore, multi-period and multi-paradigm models have also been proposed in order to plan and optimize the energy system and components for a long time planning horizon (Hodge, Huang, Siirola, Pekny, & Reklaitis, 2011; Zhang, Liu, Ma, Li, & Ni, 2012; Zhang, Liu, Ma, & Li, 2013). Recently, Soroush and Chmielewski (2013) have presented an overview of the state of the art and the current process systems opportunities in power generation, storage and distribution.

Planning the generation of electric power is based on three different classes of decisions defined according to the length of the planning time horizon: long-term decisions (capacity, type and number of power generators); medium term decisions (scheduling of the existing units); short-term decisions (programming of the power that each committed unit must produce to meet the real-time electricity demand). These three levels of decision are usually referred to as Power Expansion, Unit Commitment (UC) and Economic Dispatch, respectively. The UC problem has been more widely studied due to its practical importance (Yamin, 2004; Padhy, 2004). Moreover, this problem has diverse applications in the chemical engineering area, for example in Mitra, Grossmann, Pinto, and Arora (2012) the UC constraints were applied to air separation plants to decide when to turn on and off compressors and liquefiers.

The UC can be formulated as a mathematical programming problem using different alternative models. Implementing schedulings based on the optimal solutions of these models, may result in significant economic savings. However, solving the UC problem is very difficult. In fact, this problem is a mixed integer programming problem, linear or nonlinear, that is well known to be NP-hard due to the exponential computational time that may be required in the worst case (Nemhauser & Wolsey, 1988). A large effort has been spent over the last few decades to develop efficient methods capable of solving the UC problem for real industrial cases in practical computational times.

This paper focuses on the thermal UC problem. The solution methods proposed in the literature for solving this problem are either deterministic or heuristic. Approaches based on deterministic methods include: priority list (Senjyu, Shimabukuro, Uezato, & Funabashi, 2003), integer mathematical programming (linear

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Nomenclature

Indexes

i	unit index
t	time period index

Constants

I	total number of thermal generating units
T	length of the planning time horizon
a_i, b_i, c_i	coefficients of the fuel cost function of unit i
D_t	power load demand for time period t
R_t	spinning reserve required at time period t
p_i^L	minimum power generation of unit i
p_i^U	maximum power generation of unit i
TU_i	minimum uptime of unit i
TD_i	minimum downtime of unit i
T_i^{ini}	initial status of unit i
DR_i	ramp-down limit of unit i
UR_i	ramp-up limit of unit i
SD_i	maximum shutdown rate of unit i
SU_i	maximum startup rate of unit i
Hsc_i	hot start cost of unit i
Csc_i	cold start costs of unit i
T_i^{cold}	cold start hours of unit i
Dc_i	shut-down cost of unit i
$cost^{UP}$	upper bound for the objective function
ε^{abs}	absolute tolerance for global optimality
ε^{rel}	relative tolerance for global optimality
A_t^{opt1}	objective value of the optimal solution of problem P1 for time period t
A_t^{opt2}	objective value of the optimal solution of problem P2 for time period t
A_t^{LO}	lower bound for the number of committed units at time period t
A_t^{UP}	upper bound for the number of committed units at time period t

Variables

$u_{i,t}$	binary variable representing the on/off status of unit i at period t
$p_{i,t}$	power output of unit i in period t
$cu_{i,t}$	start-up cost of unit i in period t
$cd_{i,t}$	shut-down cost of unit i in period t
A_t	auxiliary variable for computing integer cutting planes

and nonlinear) (Cohen & Yoshimura, 1983; Rajan & Takriti, 2005; Carrión & Arroyo, 2006; Frangioni, Gentile, & Lacalandra, 2009; Zondervan, Grossmann, & de Haan, 2010; Ostrowski, Anjos, & Vannelli, 2012), dynamic programming (Ouyang & Shahidehpour, 1991), Lagrangian relaxation (Ongsakul & Petcharaks, 2004; Frangioni, Gentile, & Lacalandra, 2011; Dieu & Ongsakul, 2011) and other decomposition techniques (Habibollahzadeh & Bubenko, 1986; Niknam, Khodaei, & Fallahi 2009). However, few of these proposed methods guarantee global optimality. As for heuristic approaches, the most widely used are: artificial neural networks (Sasaki, Watanabe, Kubokawa, Yorino, & Yokoyama, 1992), genetic algorithms (Kazarlis, Bakirtizis, & Petridis, 1996; Swarup & Yamashiro, 2002), evolutionary programming (Juste, Kita, Tanaka, & Hasegawa, 1999; Chen & Wang, 2002), simulated annealing (Simopoulos, Kavatza, & Vournas, 2006), fuzzy systems (El-Saadawi, Tantawi, & Tawfik, 2004), particle swarm optimization (Ting, Rao, & Loo, 2006; Oñate Yumbla, Ramirez, & Coello Coello, 2008), tabu search (Mantawy, Abdel-Magid, & Selim, 1998) and hybrid methods (Cheng, Liu, & Liu, 2000; Mantawy, Abdel-Magid, & Selim, 1999).

Yamin (2004), Padhy (2004) and Sen and Kothari (1998) give complete reviews for contributions on deterministic and heuristic methodologies for solving the UC problem. Nevertheless, the methods proposed so far are not always able to solve real world problems to optimality in acceptable computational times.

In this paper a new deterministic optimization approach is proposed for the thermal UC problem. The problem addressed can be stated as follows: given a number of thermal power generators (differing in their operating and production characteristics) and a specified time-variant demand over the planning time horizon, determine for each unit the start-up and shut-down schedules and the power production, in order to minimize the operational costs while meeting demand.

The mathematical model is a convex mixed-integer quadratic programming problem (MIQP) for which a Branch and Cut method is proposed that takes advantage of the characteristics of the UC problem.

The paper is organized as follows. Section 2 presents a detailed description of the mathematical formulation for the thermal UC problem. Section 3 describes the proposed deterministic optimization approach, outlining in Section 3.1 the steps to construct the proposed integer cutting planes. In Section 3.2 a particular implementation of the general Branch-and-Bound framework is described. Section 4 presents computational tests with the proposed optimization approach. In Section 4.1, the performance of the proposed integer cutting planes is illustrated. In Section 4.2 three application examples are presented with the proposed technique, compared with other deterministic methods, and alternative mathematical formulations. Finally, Section 5 draws the general conclusions.

2. Mathematical problem formulation

The thermal UC problem is formulated as the following MIQP model. Consider a set of I thermal generating units and a specified time-varying demand over T time periods defining the planning time horizon, with the units being indexed with $i=1,\ldots,I$ and the time periods with $t=1,\ldots,T$. The mathematical programming model involves $I \times T$ binary variables: $u_{i,t}$; and $3(I \times T)$ continuous variables: $p_{i,t}$, $cu_{i,t}$ and $cd_{i,t}$; for $i=1,\ldots,I$ and $t=1,\ldots,T$.

The objective function to be minimized is the operating cost, which includes fuel consumption calculated by a quadratic function with fixed charges, and fixed start-up and shut-down costs:

$$\min\ cost = \sum_{i=1}^{I}\sum_{t=1}^{T}[(a_i u_{i,t} + b_i p_{i,t} + c_i p_{i,t}^2) + cu_{i,i} + cd_{i,t}] \tag{1}$$

The constraints to be satisfied are given by (2)–(20).

Satisfying power demand for each time period:

$$D_t \le \sum_{i=1}^{I} p_{i,t} \quad t = 1, \ldots, T \tag{2}$$

Spinning reserve is guaranteed by the available capacity of active units:

$$D_t + R_t \le \sum_{i=1}^{I} p_i^U u_{i,t} \quad t = 1, \ldots, T \tag{3}$$

The generation power limits of each unit at each time period are given by:

$$u_{i,t} p_i^L \le p_{i,t} \le p_i^U u_{i,t} \quad i = 1, \ldots, I; \quad t = 1, \ldots, T \tag{4}$$